

# Risk Premium Report 2011

## Selected Pages and Examples

This document is an excerpt of the 2011 *Risk Premium Report*, and includes an overview of the methodologies employed in performing the analysis required for the *Size Study*, *Risk Study*, and *High-Financial-Risk Study* that constitute the *Duff & Phelps Risk Premium Report*. The excerpt also includes a limited number of examples demonstrating how the

*Risk Premium Report's* size premia and risk premia data can be used to estimate cost of equity capital (more examples are available in the complete *Report*). The excerpt does not include the size and risk premia data exhibits that are available in the full version of the *Risk Premium Report*.

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# Publication information/Disclaimer/ Purchasing information

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## 2011 Duff & Phelps Risk Premium Report

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\* The *Duff & Phelps Risk Premium Report* is intended to be used as a companion publication to the web-based *Duff & Phelps Risk Premium Calculator*. Exclusive distribution for the *Calculator* is through BVR.

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# Introduction

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## History of the Duff & Phelps Risk Premium Report

In 1990, Roger Grabowski began closely studying the relationship between company size and stock returns.<sup>1</sup> Grabowski's early research focused on size as measured by market capitalization, but quickly advanced to two additional areas of inquiry: whether stock returns were predicted by measures of size other than market capitalization, and whether stock returns were predicted by fundamental risk measures based on accounting data. To investigate these questions, in 1992 Grabowski, working with a colleague<sup>2</sup>, contracted with the Center for Research in Security Prices (CRSP) at the University of Chicago to build a database that combined stock prices, number of shares, and dividend data from the CRSP database with accounting and other data from the Standard & Poor's *Compustat* database.

What they found was that as size decreases, or risk increases (as measured by fundamental accounting data), returns tend to increase (and vice versa). Thereafter, they published a series of articles reporting their findings, culminating with a seminal 1996 article and a subsequent article in 1999 which together served as the foundation of the *Duff & Phelps Risk Premium Report*.<sup>3</sup>

Now in its 16th year of publication, the *Duff & Phelps Risk Premium Report* continues to be at the forefront in providing comprehensive valuation methodology and data.

The *2011 Duff & Phelps Risk Premium Report* ("2011 Report") includes data available through December 31, 2010, and should be used for calendar year 2011 valuations.

## Roger J. Grabowski, ASA, Author

Managing Director, Duff & Phelps

## James P. Harrington, Editor

Director, Duff & Phelps

## Recent Changes and Additions

In 2011 we are pleased to introduce the new web-based *Duff & Phelps Risk Premium Calculator*.<sup>4</sup> The *Risk Premium Calculator* is based on the same trusted data and analysis published in the *Risk Premium Report* since 1996.

### Calculator Features

- Anytime, anywhere access at [www.bvmarketdata.com/DP.RPC](http://www.bvmarketdata.com/DP.RPC)
- Complete historical database of risk premia and size premia data (1996–2011)
- Automatic output
  - Executive Summary of COE estimates, including CAPM, Buildup, and unlevered COE
  - Microsoft Excel output of all underlying values and calculations
- Easy to use / Saves time

For more information about the new *Risk Premium Calculator*, please see "New in 2011: The Duff & Phelps Risk Premium Calculator (web-based)" on page 67.

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<sup>1</sup> Roger Grabowski, ASA, is a managing director in the Duff & Phelps Chicago office and part of the firm's Valuation Advisory Service practice, and co-author with Dr. Shannon Pratt of *Cost of Capital: Applications and Examples*, 4th Edition (John Wiley & Sons, 2010).

<sup>2</sup> David King, CFA, is National Technical Director of Valuation Services at Mesirow Financial Consulting, LLC. The research began when both he and Roger Grabowski were at Price Waterhouse, predecessor firm to PricewaterhouseCoopers.

<sup>3</sup> Roger J. Grabowski and David King, "New Evidence on Size Effects and Equity Returns", *Business Valuation Review* (September 1996, revised March 2000), & Roger J. Grabowski and David King, "New Evidence on Equity Returns and Company Risk", *Business Valuation Review* (September 1999, revised March 2000).

<sup>4</sup> Please note that in 2011, the *Duff & Phelps Risk Premium Calculator* is available exclusively through Business Valuation Resources (BVR). For more information about the *Calculator*, visit: [www.bvresources.com/dp](http://www.bvresources.com/dp) or call toll free 1-(888)-287-8258.

# Introduction

## Who Should Use the Duff & Phelps Risk Premium Report

The *Risk Premium Report* is designed to assist financial professionals in estimating the cost of equity capital (“cost of equity”, or “COE”) for a subject company. The risk premia calculated in the *Report* can be used to develop COE estimates using both the build-up method and the Capital Asset Pricing Model (CAPM).

In addition to the traditional professional valuation practitioner, the *Risk Premium Report*, and the accompanying web-based *Risk Premium Calculator* (“Calculator”), are designed to serve the needs of:

- Corporate finance officers for pricing or evaluating mergers and acquisitions, raising private or public equity, property taxation, and stakeholder disputes.
- Investment bankers for pricing public offerings, mergers and acquisitions, and private equity financing.
- CPAs who deal with either valuation for financial reporting or client valuations issues.
- Judges and attorneys who deal with valuation issues in mergers and acquisitions, shareholder and partner disputes, damage cases, solvency cases, bankruptcy reorganizations, property taxes, rate setting, transfer pricing, and financial reporting.

## Appropriate Use of the Duff & Phelps Risk Premium Report

The information and data in the *Risk Premium Report* (and in the online *Risk Premium Calculator*)<sup>5</sup> is primarily designed to be used to develop cost of equity capital (COE) estimates for large majority of companies that are fundamentally healthy, and for which a “going concern” assumption is appropriate. “High-financial-risk” (i.e. “distressed”) companies are excluded from the base dataset and analyzed separately.

Because financial services companies are excluded from the base set of companies used to develop the analyses presented in the *Report*, the *Report* (and the online *Calculator*) should not be used to estimate cost of equity for financial services companies. Financial services companies include those companies in finance, insurance, or real estate (i.e. companies with an SIC Code that begins with “6”).

<sup>5</sup> The *Duff & Phelps Risk Premium Calculator* is available exclusively through Business Valuation Resources (BVR). For more information about the *Calculator*, visit: [www.bvresources.com/dp](http://www.bvresources.com/dp) or call toll free 1-(888)-287-8258.

# How the 2011 Report is Organized

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The *Risk Premium Report* is divided into two main sections: a methodology section, followed by a data exhibits section.

## First Section: Methodology

The first section features a discussion of the data and methodology used to create the portfolios used to perform the analysis in the *Report*, as well as an overview of the *Size Study*, *Risk Study*, and *High-Financial-Risk Study* (with examples of how to use each of these studies to estimate cost of equity capital). Also included are Appendices and a Glossary of terms:

- **Portfolio Methodology:** A discussion of the data and methodology used to create the portfolios used to perform the analysis in the *Report*.
- **Size Study:** Analyzes the relationship between equity returns and company size, using up to eight measures of company size (i.e. "size measures").
- **Risk Study:** Analyzes the relationship between equity returns and accounting-based fundamental risk measures.
- **High-Financial-Risk Study:** Analyzes the relationship between equity returns and high-financial-risk, as measured by the Altman z-Score.
- **Appendices:** Definitions of *Compustat* data items, and a summary of changes from previous versions of the *Report* (over time).
- **Glossary:** A list of important terms with accompanying definitions.

# How the 2011 Report is Organized

## Second Section: Data Exhibits

The second section features the data exhibits in which the various risk and size premia used to estimate cost of equity capital are found.

Each of the three *Studies* (*Size Study*, *Risk Study*, and *High-Financial-Risk Study*) discussed in the Methodology section have corresponding data exhibits (A, B, D, or H), as illustrated in Figure 1.

**Figure 1: *Size Study*, *Risk Study*, *High-Financial-Risk Study* and Corresponding Exhibits**



The risk premia in the A, B, D, and H exhibits can be used to develop cost of equity capital estimates using both the buildup method and the capital asset pricing model (CAPM). In addition, a “link” between the *Size Study* and *Risk Study* is provided in the C exhibits.

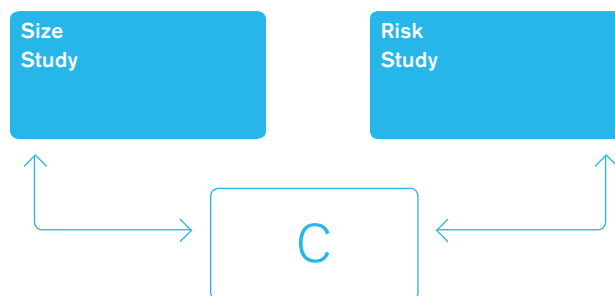
- **Exhibits A-1 through A-8:** The A exhibits provide risk premia over the risk free rate in terms of the total effect of *market risk* and *size risk* for 25 portfolios ranked by eight alternative measures of size ( $RP_{m+s}$ ).
- **Exhibits B-1 through B-8:** The B exhibits provide risk premia over CAPM (“size premia”) in terms of *size risk* for 25 portfolios ranked by eight alternative measures of size ( $RP_s$ ).
- **Exhibits C-1 through C-8:** The C exhibits provide a “link” between the *Size Study* and the *Risk Study*. These exhibits can be used to compare a subject company's fundamental risk characteristics to the fundamental risk characteristics of portfolios made up of similarly-sized companies.

For example, the C exhibits can help to answer whether the subject company is more or less profitable (as measured by operating margin) than similarly-sized companies, or whether the subject company's earnings are more or less volatile (as measured by coefficient of variation of operating margin and coefficient of variation of ROE) than similarly-sized companies.

In the former case, the less profitable the subject company is, all other things held the same, the riskier it is (and vice versa). In the latter two cases (which are measures of earnings volatility), the more volatile a company's earnings are, all other things held the same, the less predictable they are, and thus the riskier the company is (and vice versa).

This is an important capability because this type of analysis can be used as an indication that a company-specific risk adjustment is needed (either upward or downward).

**Figure 2: The C Exhibits – *Size Study* and *Risk Study* “Link”**



- **Exhibits D-1, D-2, and D-3:** The D exhibits provide risk premia over the risk free rate in terms of the total effect of *market risk* and *company-specific risk* for 25 portfolios ranked by three alternative measures of fundamental risk ( $RP_{m+u}$ ).
- **Exhibits H-A, H-B, and H-C:** The H exhibits provide “high-financial-risk” premia for portfolios ranked by Altman z-Score<sup>6</sup>. These premia may be used in both buildup and CAPM estimates of cost of equity capital if the individual analyst has determined that the subject company is “high-financial-risk.”<sup>7</sup> Exhibit H-A is the high-financial-risk equivalent of the A exhibits, Exhibit H-B is the high-financial-risk equivalent of the D exhibits, and Exhibit H-C is the high-financial-risk equivalent of the C exhibits.

<sup>6</sup> Altman z-Score is an accounting-data-based method designed to assess financial condition and developed originally for assessing the likelihood of bankruptcy.

<sup>7</sup> The decision to apply a high-financial-risk premium is ultimately dependent on the analyst's professional judgment, based upon the analyst's detailed knowledge of the subject company.



# Portfolio Methodology

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## Data Sources

The universe of companies used to perform the analyses presented in the *Risk Premium Report* is comprised of those companies that are found in both the Center for Research in Security Prices (CRSP) database at the University of Chicago Booth School of Business and Standard and Poor's *Compustat* database.

## Historical Time Period Used

In the 2011 *Risk Premium Report*, risk premia and other useful statistics are developed using historical equity returns (from CRSP), and fundamental accounting data (from *Compustat*) over the period 1963 through 2010.

The *Compustat* database was established in 1963. While *Compustat's* fundamental accounting data is available for some companies going back to the 1950s, this earlier data consists only of the back histories for companies that were added to *Compustat* in 1963 or later. The *Report's* analysis begins with 1963 data in order to avoid the obvious selection bias that would result from using the earlier data.

For each year covered in the *Report*, financial data for the fiscal year ending no later than September of the previous year is considered. For example, when assigning a company to a portfolio to calculate returns for calendar year 1995, financial data through the latest fiscal year ending September 1994 or earlier is considered (depending on when the company's fiscal year ended).

## Exclusions

After identifying a universe of companies that are in both the CRSP and *Compustat* databases, the following types of firms are excluded:

- American Depositary Receipts (ADRs)
- Non-operating holding companies
- Financial service companies (SIC code 6)

Financial service companies (those companies in finance, insurance, or real estate) are excluded because some of the financial data used in the *Report* is difficult to apply to companies in the financial sector (for instance, "sales" at a commercial bank). In addition, financial service companies tend to support a much higher ratio of debt to equity than do other industries, and so including them in with non-financial firms may be an "apples to oranges" comparison that could lead to improperly skewed results. Moreover, companies in the financial services sector were poorly represented during the early years of the *Compustat* database.

It should be noted that since financial service companies are excluded from the set of companies used to perform the analyses presented in the *Report*, these results should not be used by an analyst estimating the cost of equity capital (COE) for a financial services company.

Altogether, companies are excluded (or segregated) based upon their past financial performance or trading history in the *Risk Premium Report*. It should be noted that alternative analyses in which no companies were excluded or segregated on the basis of past financial performance or trading history have been performed (that is, using all available non-financial companies). The results are similar, but these exclusions are maintained as a precaution against the possibility of introducing a bias in favor of the size effect (to the extent that such companies tend to have low market values).

# Portfolio Methodology

## Unseasoned Companies

The small cap universe may consist of a disproportionate number of start-up companies and recent initial public offerings. These “unseasoned” companies may be inherently riskier than companies with a track record of viable performance. For this reason (for each year since 1963), we screen the universe of companies to exclude companies with any of the following characteristics<sup>8</sup>:

- Companies lacking 5 years of publicly traded price history
- Companies with sales below \$1 million in any of the previous five fiscal years
- Companies with a negative 5-year-average EBITDA (earnings before interest, taxes, depreciation and amortization) for the previous five fiscal years
- Companies not listed on one of the major US stock exchanges (NYSE, AMEX or NASDAQ)

The set of companies remaining after this screen are seasoned companies in that they have been traded for several years, have been selling at least a minimal quantity of product, and have been able to achieve a degree of positive cash flow from operations.

## High-Financial-Risk Study

After eliminating companies with the characteristics described previously, the remaining companies are screened again to exclude companies with any of the following characteristics<sup>9</sup>:

- Companies that Standard & Poor's has identified in the *Compustat* database as in bankruptcy or in liquidation,
- Companies with a “5-year average net income available to common equity” less than zero for the previous five years (either in absolute terms or as a percentage of the book value of common equity),
- Companies with “5-year-average operating income” (sales minus cost of goods sold plus selling, general and administrative expenses plus depreciation) less than zero for the previous five years (either in absolute terms or as a percentage of net sales),

- Companies with negative book value of equity at any one of the company's previous five fiscal year-ends, and
- Companies with a debt-to-total capital ratio of more than 80%, (debt is measured in book value terms, and total capital is measured as book value of debt *plus* market value of equity).

The companies excluded in this screen are set aside and analyzed separately in the *High-Financial-Risk Study*.

This screen is performed in an effort to isolate the effects of high-financial-risk. Otherwise, the results might be biased for smaller companies to the extent that highly leveraged and financially distressed companies tend to have both high returns and low market values.

It is possible to imagine companies that don't have any of these characteristics, but could still be classified as high-financial-risk (i.e. “distressed”), and it is also possible to imagine companies which do have one or more of these characteristics but are *not* distressed. Nevertheless, the resulting high-financial-risk database is composed largely of companies whose financial condition is significantly inferior to the average, financially “healthy” public company.

## Exclusions are Based on Past Information

The exclusion of companies is based on their *past* financial performance or trading history as of the time that the portfolios are formed for any given year over the 1963–2010 time horizon. For example, to form portfolios for 1963, company data for the previous 5 fiscal years (prior to September 1962) is considered. This procedure is repeated for *each* year from 1963 through the latest available year for *each* of the eight measures of size examined in the *Size Study*, and for *each* of the three measures of fundamental risk examined in the *Risk Study*. All of the previously discussed exclusions are therefore not based on any unusual foresight on the part of hypothetical investors in these portfolios, but are based on information that was already “history” at the time the portfolios were created.

<sup>8</sup> The number of companies eliminated in this screen varies from year to year.

<sup>9</sup> The number of companies eliminated in this screen varies from year to year. These companies represented up to 25% of the data set in recent years, but less than 5% in 1963. Certain technical changes in methodology have resulted in a greater number of companies falling into the high-financial-risk database than in versions of this study published prior to 2000.

# Portfolio Methodology

## Portfolio Creation

After excluding unseasoned and segregating high-financial-risk companies, the result is a base set of companies that is used for the analyses performed in both the *Size Study* and the *Risk Study*.

The major difference between the two studies is that the portfolios presented in the *Size Study* are ranked by eight alternative measures of size, from largest (Portfolio 1) to smallest (Portfolio 25), while the portfolios presented in the *Risk Study* are ranked by three accounting-based measures of fundamental risk, from lowest risk (Portfolio 1) to highest risk (Portfolio 25). The smallest size/highest risk portfolios tend to have the highest returns.

Other than that difference, portfolio formation in the *Size Study* and *Risk Study* is a very straightforward process. This process is described in the following sections.

## Size Study Portfolio Creation

To perform the analysis required for the *Size Study*, 25 portfolios are created from companies that are similarly-sized, with Portfolio 1 made up of the largest companies and Portfolio 25 made up of the smallest companies. The equity returns for each of the 25 portfolios returns are calculated using an equal-weighted average of the companies in the portfolio, and these returns are then used to calculate risk premia (and other useful information and statistics) for each.

“Size” is defined by the traditional size measure, market value of common equity (i.e. “market capitalization”), as well as seven additional size measures:

- 1) Market value of common equity
- 2) Book value of common equity
- 3) 5-year average net income
- 4) Market value of invested capital (MVIC)
- 5) Total Assets
- 6) 5-year average EBITDA<sup>10</sup>
- 7) Sales
- 8) Number of employees

The first step is to determine portfolio breakpoints for the 25 portfolios. Portfolio breakpoints are the upper and lower “boundaries” of each portfolio, represented by the largest and smallest New York Stock Exchange (NYSE) company, respectively, in each of the 25 portfolios. For example, to determine the breakpoints for the 25 portfolios ranked by “Total Assets”, all of the companies in the base set that are traded on the NYSE are ranked from largest (in total assets) to smallest (in total assets), and then divided into 25 equally populated portfolios.

<sup>10</sup> Earnings before interest, income taxes, depreciation and amortization.

# Portfolio Methodology

Once portfolio breakpoints are determined, companies from the NYSE Amex Equities (formerly the American Stock Exchange, or AMEX)<sup>11</sup> universe and the NASDAQ universe are added to the appropriate portfolio, depending on their size with respect to the breakpoints.<sup>12</sup> Since NYSE Amex Equities and NASDAQ companies are generally small relative to NYSE companies, their addition to the data set produces portfolios that are more heavily populated at the “small cap” end of the spectrum.<sup>13</sup>

All portfolios are rebalanced annually, so this process is completed for *each* year from 1963 to the most recent available year, and for *each* of the eight measures of size. This results in the creation of 25 portfolios for each of the eight size measures, a total of 200 (8 x 25) unique portfolios for each year from 1963 to present, each ranked from largest to smallest by each respective size measure.<sup>14</sup>

## Risk Study Portfolio Creation

To perform the analysis required for the *Risk Study*, 25 portfolios are created from companies that have similar accounting-data-based fundamental risk characteristics, with Portfolio 1 made up of companies with the lowest fundamental risk, and Portfolio 25 made up of companies with the highest fundamental risk.

The equity returns for each of the 25 portfolios returns are calculated using an equal-weighted average of the companies in the portfolio, and these returns are then used to calculate risk premia (and other useful information and statistics) for each.

“Fundamental Risk” is defined by the following three alternative measures (the first is a measure of profitability; the latter two are measures of earnings variability):

- 1) Operating margin
- 2) Coefficient of variation in operating margin
- 3) Coefficient of variation in return on equity

As in the *Size Study*, the first step is to determine portfolio breakpoints for the 25 portfolios. Using “Operating Margin” as an example, all companies in the base set that are traded on the New York Stock Exchange (NYSE) are ranked from lowest fundamental risk (highest operating margin) to highest fundamental risk (lowest operating margin), and then divided into 25 equally populated portfolios.

Once portfolio breakpoints are determined, companies from the NYSE Amex Equities universe and the NASDAQ universe are added to the appropriate portfolio, depending on their fundamental risk with respect to the breakpoints.

Since all portfolios are rebalanced annually, this process is followed for *each* year from 1963 to the most recent available year, for *each* of the three measures of fundamental risk. This results in the creation of 25 portfolios for each of the three fundamental risk measures, a total of 75 (3 x 25) unique portfolios for each year from 1963 to present, each ranked from lowest risk to highest risk for each respective measure of fundamental risk.<sup>15</sup>

<sup>11</sup> On October 1 2008, NYSE Euronext acquired the American Stock Exchange (AMEX). Post merger, the AMEX equities business was branded “NYSE Alternext US”. NYSE Alternext US was subsequently re-branded “NYSE Amex Equities”, which remains its name today.

<sup>12</sup> NYSE Amex Equities data is available after 1962 and NASDAQ data is available after 1972.

<sup>13</sup> Some readers may ask why NYSE breakpoints are used rather than ranking the entire NYSE/NYSE Amex/NASDAQ universe. The consistent use of NYSE breakpoints avoids an apples-to-oranges mixing of pre-1972 (pre-NASDAQ) ranking criteria with post-1972 ranking criteria. Otherwise, “average” NASDAQ companies (in recent years) would be assigned to portfolios that contain much larger “average” NYSE companies (in earlier years) when calculating average returns for the mid-sized portfolios over the full sample period. The only logical alternatives are either to adopt the NYSE breakpoint approach or to exclude NASDAQ companies altogether.

<sup>14</sup> In the *2011 Report*, this represents 8 size measures x 25 portfolios x 48 years (1963–2010) = 9,600 unique portfolio formations to perform the analysis presented in the *Size Study*.

<sup>15</sup> In the *2011 Report*, this represents 3 measures of fundamental risk x 25 portfolios x 48 years (1963–2010) = 3,600 unique portfolio formations to perform the analysis presented in the *Risk Study*.

# Portfolio Methodology

## Correcting for Delisting Bias

Previous evidence indicated that the CRSP database omits delisting returns for a large number of companies for the month in which a company is delisted from an exchange.<sup>16</sup> Data was collected for a large number of companies that had been delisted for performance reasons (e.g. bankruptcy, or insufficient capital) and found that investors incurred an average loss of about 30% after delisting.

While CRSP has improved their database by reducing the number of companies for which it omits delisting returns, we incorporate this evidence into our rate of return calculations by applying a 30% loss in the month of delisting in all cases where the delisting return is missing and for which CRSP identified the reason for delisting as “performance related”. As an additional precaution, this adjustment is also applied in all cases in which the reason for delisting was identified by CRSP as “unknown”.<sup>17</sup>

## Size and Risk Rankings are Based on Past Information

The ranking of companies based on size and fundamental risk does not imply any unusual foresight on the part of hypothetical investors in these portfolios – the data used is as of the *beginning* of each year, and thus was already “history” at the time the portfolios are formed.

<sup>16</sup> “The Delisting Bias in CRSP Data,” Tyler Shumway, *Journal of Finance* (March 1997).

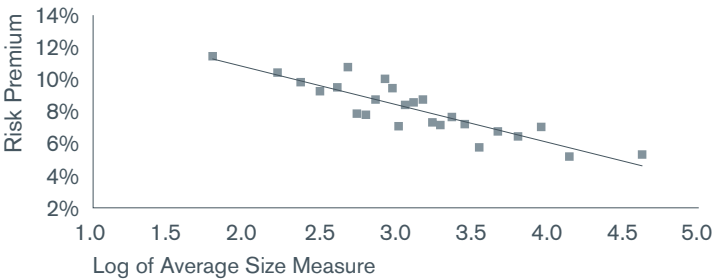
<sup>17</sup> This approach is consistent with updates that we have published since 1998. More recent evidence suggests that the average “delisting” loss is less than Shumway’s original estimate. For more information about CRSP and CRSP delisting returns, visit [www.CRSP.com](http://www.CRSP.com).

# Using the 2011 Report

## Using “Smoothed” Premia versus Using “Average” Premia

The difference between average risk premia and smoothed risk premia is illustrated in Graph 1a and Graph 1b.

Graph 1a: Average Risk Premia for 25 Portfolios with a Best Fit Line Added

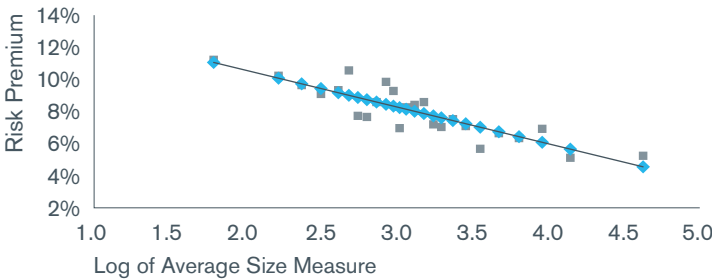


In Graph 1a, the square gray points represent a scatter plot of size (on the horizontal “x” axis), and the average risk premium (for each of 25 size-ranked portfolios, on the vertical “y” axis).<sup>18</sup> Note that as size increases from left to right, the risk premium tends to decrease (and vice versa).

The “best fit” line is the straight (“smooth”) line in Graph 1a. Using regression analysis, an equation for the best fit line can be calculated, and this equation can be used to estimate “smoothed” risk premia for the 25 portfolios based upon the average size measure of each portfolio.

A scatter plot of risk premia smoothed in this fashion and the log of the size measures will necessarily fall on the best fit line (smoothed risk premia are represented by the blue diamonds in Graph 1b).

Graph 1b: Smoothed Risk Premia



<sup>18</sup> In this example, “risk premium” is used generically. The same statistical techniques described in this example are used to calculate smoothed “risk premia over the risk free rate” (the A exhibits) and “risk premia over CAPM” (the B exhibits), as well as smoothed unlevered premia (the C Exhibits).

## Using the 2011 Report

Smoothing the premia essentially averages out the somewhat scattered nature of the raw average premia. The “smoothed” average risk premium is generally the most appropriate indicator for most of the portfolio groups. It should be noted, however, that at the largest-size and smallest-size ends of the range, the average historical risk premiums may tend to jump off of the smoothed line, particularly for the portfolios ranked by size measures that incorporate market capitalization (exhibits A-1 and A-4). Because the size measure is expressed in logarithms, this is equivalent to the change in risk premium given the percentage change in the size of the companies from portfolio to portfolio.

Smoothed risk premia are found in the data exhibits. For example, in Figure 3 the smoothed risk premium over the risk free rate for Portfolio 24 in Exhibit A-2 is 11.16 percent.<sup>19</sup>

In this example, the 11.16 percent risk premium is calculated based upon the average book value of equity of companies in Portfolio 24 (\$166 million). However, the subject company's size rarely exactly matches the average size of companies in the guideline portfolio. In the next section, how to interpolate an “exact” risk premium value when the subject company's size is “in between” guideline portfolios is explained.

### Using the Regression Equation Method to Calculate Interpolated Risk Premia Between Guideline Portfolios

The *Risk Premium Report* provides two ways for users to match their subject company's size (or risk) characteristics with the appropriate smoothed premia: the “guideline portfolio” method, and the “regression equation” method. When the subject company's size (or risk) does not exactly match the average company size (or risk) of the guideline portfolio, the regression equation method is a straightforward and easy way to interpolate between the guideline portfolios.

**Figure 3: Smoothed Premia in Exhibit A-2**  
**Companies Ranked by Book Value of Equity**  
 Historical Equity Risk Premium: Average Since 1963  
 Data for Year Ending December 31, 2010

Portfolio Rank by Size	Average Book Val. (\$mils.)	Log of Average Book Val.	Number as of 2010	Beta (SumBeta) Since '63	Standard Deviation of Returns	Geometric Average Return	Arithmetic Average Return	Arithmetic Average Risk Premium	Smoothed Average Risk Premium	Average Debt/MVIC
1	39,141	4.59	37	0.81	15.98%	10.51%	11.98%	5.08%	4.42%	23.89%
2	12,811	4.11	31	0.85	16.44%	10.68%	12.21%	5.30%	5.80%	28.38%
3	8,823	3.95	31	0.92	16.60%	11.82%	13.42%	6.52%	6.26%	29.11%
<div> <div>←</div> <div>///</div> <div>→</div> </div>										
24	166	2.22	110	1.27	25.04%	14.79%	18.05%	11.15%	11.16%	23.61%
25	60	1.78	374	1.27	26.09%	15.41%	18.89%	11.98%	12.41%	24.19%

<sup>19</sup> The A exhibits include “risk premia over the risk free rate” which are added to a risk free rate to estimate cost of equity capital using the buildup method. Please refer to the individual examples provided for these models for more information and examples.

# Using the 2011 Report

For example, if the subject company's book value of equity in the previous example was \$114 million, one would expect the smoothed average size premium to fall somewhere between 11.16 percent (the smoothed size premium for guideline Portfolio 24) and 12.41 percent (the smoothed size premium for guideline Portfolio 25). To calculate the "exact" smoothed premium between guideline portfolios, use the regression equations provided in each of the exhibits (please note that there is a different equation for each of the exhibits). For example, in Figure 4 the regression equation provided for Exhibit A-2 is<sup>20</sup>:

$$\text{Smoothed Premium} = 17.475\% - 2.843\% \times \log(\text{Book Value})$$

Inserting the subject company's market cap of \$114 million into this equation results in an "exact" smoothed premium of 11.63%:

$$\begin{aligned}\text{Smoothed Premium} &= 17.475\% - 2.843\% \times \log(\$114 \text{ million}) = \\ 11.63\% &= 17.475\% - 2.843\% \times 2.06\end{aligned}$$

## Guideline Portfolio Method or Regression Equation Method?

The major difference between the "guideline portfolio" and the "regression equation" methods is that with the guideline method, one accepts the smoothed average risk premium published in the report (calculated using the average size in each of the 25 guideline portfolios), while with the regression equation method, one can calculate an "exact" interpolated value between the guideline portfolios. For this reason, although the guideline portfolio is simpler and more direct, the more flexible regression equation method is the suggested method in most cases.

In practice this approach generally produces results that are very similar to those of the guideline portfolio approach presented above (unless one is extrapolating to a company that is much smaller than the average size for the 25th portfolio).

**Figure 4: Location of "Regression Method" Equation in the Data Exhibits**

Companies Ranked by Book Value of Equity  
Historical Equity Risk Premium: Average Since 1963  
Data for Year Ending December 31, 2010

Portfolio Rank by Size	Average Book Val. (\$mls.)	Log of Average Book Val.	Number as of 2010	Beta (SumBeta) Since '63	Standard Deviation of Returns	Geometric Average Return	Arithmetic Average Return	Arithmetic Average Risk Premium	Smoothed Average Risk Premium	Average Debt/MVIC
1	39,141	4.59	37	0.81	15.98%	10.51%	11.98%	5.08%	4.42%	23.89%
2	12,811	4.11	31	0.85	16.44%	10.68%	12.21%	5.30%	5.80%	28.38%
3	8,823	3.95	31	0.92	16.60%	11.82%	13.42%	6.52%	6.26%	29.11%
4	6,398	3.81	31	0.91	16.58%	11.25%	12.78%	5.87%	6.66%	28.50%
5	4,540	3.66	37	1.02	18.59%	11.26%	13.21%	6.31%	7.08%	26.89%
6	3,433	3.54	30	1.02	18.45%	12.01%	13.89%	6.99%	7.42%	26.37%
7	2,868	3.46	30	1.05	20.33%	11.85%	14.04%	7.13%	7.65%	24.98%
8	2,323	3.37	39	1.07	18.97%	12.42%	14.41%	7.51%	7.91%	24.85%
9	1,969	3.29	32	1.12	20.68%	12.50%	14.82%	7.91%	8.11%	25.15%
10	1,704	3.23	34	1.06	19.27%	13.07%	15.06%	8.15%	8.29%	25.56%
11	1,497	3.18	34	1.08	20.21%	13.01%	15.22%	8.31%	8.45%	26.77%
12	1,304	3.12	41	1.09	20.59%	14.17%	16.42%	9.52%	8.62%	25.69%
13	1,155	3.06	38	1.10	21.58%	14.23%	16.70%	9.80%	8.77%	25.53%
14	1,059	3.02	38	1.14	20.85%	12.86%	15.13%	8.23%	8.88%	23.85%
15	939	2.97	45	1.11	21.22%	14.91%	17.29%	10.38%	9.02%	23.90%
16	812	2.91	38	1.22	24.33%	15.66%	18.61%	11.70%	9.20%	24.16%
17	710	2.85	47	1.22	23.69%	13.30%	16.15%	9.24%	9.37%	23.28%
18	635	2.80	44	1.19	22.14%	14.57%	17.14%	10.23%	9.51%	23.97%
19	548	2.74	55	1.24	21.36%	12.66%	15.12%	8.21%	9.69%	23.84%
20	464	2.67	59	1.21	22.02%	13.47%	15.99%	9.09%	9.89%	23.57%
21	380	2.58	87	1.22	21.24%	15.43%	17.86%	10.96%	10.14%	23.47%
22	303	2.48	71	1.25	23.17%	14.01%	16.88%	9.98%	10.42%	23.98%
23	230	2.36	86	1.26	23.26%	14.43%	17.22%	10.32%	10.76%	24.14%
24	166	2.22	110	1.27	25.04%	14.79%	18.05%	11.15%	11.16%	23.61%
25	60	1.78	374	1.27	26.09%	15.41%	18.89%	11.98%	12.41%	24.19%
Large Stocks (Ibbotson SBBI data)						9.84%	11.30%	4.40%		
Small Stocks (Ibbotson SBBI data)						13.72%	16.53%	9.63%		
Long-Term Treasury Income (Ibbotson SBBI data)						6.88%	6.90%			

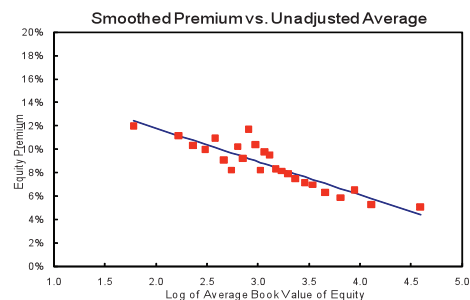
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Exhibit A-2  
Equity Risk Premium Study: Data through December 31, 2010  
Data Smoothing with Regression Analysis  
Dependent Variable: Average Premium  
Independent Variable: Log of Average Book Value of Equity

Regression Output:  
Constant 17.475%  
Std Err of Y Est 0.876%  
R Squared 81%  
No. of Observations 25  
Degrees of Freedom 23  
  
X Coefficient(s) -2.843%  
Std Err of Coef. 0.285%  
t-Statistic -9.98

$$\text{Smoothed Premium} = 17.475\% - 2.843\% \times \log(\text{Book Value})$$



$$\text{Smoothed Premium} = 17.475\% - 2.843\% \times \log(\text{Book Value})$$

<sup>20</sup> The term "log" is the base 10 logarithm. The base 10 log of 114 is 2.06. To calculate a base 10 log in Microsoft Excel, use =log(size measure). Remember that the logarithmic relationship is base-10, and that the financial size data is in millions of dollars, such that the log of \$10 million is log (10), and not log (10,000,000).



# Using the 2011 Report

## Using the Regression Equation Method to Calculate Interpolated Risk Premia for Smaller Companies.

Sometimes one needs to estimate the cost of equity capital for a company that is significantly smaller than the average company size of even the smallest of the *Report's* 25 portfolios. In such cases, it may be appropriate to extrapolate the risk premium to smaller sizes using the regression equation method. Table 1 summarizes the size of companies by each of the eight alternative size measures, by percentile ranking.<sup>21</sup>

For example, the 95th percentile of size for book value of equity is \$123.477 million, which means that 95 percent of the companies in Portfolio 25 have book value of equity that is *less* than \$123.477 million (alternatively, this means that 5 percent of the companies in Portfolio 25 have book value of equity that is *greater* than \$123.477 million). Or, looking now to the 5th percentile, 5 percent of the companies in Portfolio 25 have book value of equity that is *less* than \$9.563 million (alternatively, this means that 95 percent of the companies in Portfolio 25 have book value of equity that is *greater* than \$9.563 million).

As a general rule, extrapolating a statistical relationship far beyond the range of the data used in the statistical analysis is not recommended. However, extrapolations for companies with size characteristics that are within the range of companies comprising the 25th portfolio are within reason.

In some cases the size of the subject company may be equal to or greater than the smallest size of the companies included in the 25th portfolio for one size measure (e.g., sales), but less than the smallest size of the companies included in the 25th portfolio for another size measure (e.g., 5-year average income). In such cases analysts may consider *including* the size measure for sales, but *excluding* the size measure for 5-year average net income. One should never use those size measures for which the subject company's size is equal to zero or negative.

**Table 1: Size Measures of Companies that Comprise Portfolio 25, by Percentile**

	Market Value of Equity	Book Value of Equity	5-year Average Income	Market Value of Invested Capital
5th Percentile	\$8.060	<b>\$9.563</b>	\$0.495	\$10.372
25th Percentile	26.447	26.244	1.683	37.277
50th Percentile	57.626	56.114	3.654	84.184
75th Percentile	104.870	91.835	6.165	141.129
95th Percentile	152.923	<b>123.477</b>	8.247	220.326

	Total Assets	5-year Average EBITDA	Sales	Number of Employees
5th Percentile	\$16.732	\$1.829	\$17.345	13
25th Percentile	48.191	5.649	49.578	112
50th Percentile	110.834	12.388	105.445	245
75th Percentile	171.989	21.391	176.928	379
95th Percentile	252.663	31.393	241.513	524

## Size Study or Risk Study?

Analysts should use the *Size Study* if it has been determined that the risks of the subject company are comparable to the average of the portfolio companies of comparable size (e.g., comparable operating margin). One can determine the relative risk characteristics by looking at Exhibits C-1 through C-8.

But if the risk characteristics of the subject company are significantly greater or less than the portfolios comprised of companies of similar size, the *Risk Study* helps the analyst determine how much greater or less than the average expected returns should he conclude for the subject company than indicated in the Exhibits A (for build-up) or Exhibits B (for CAPM).

<sup>21</sup> The information in Table 1 was published as "Exhibit E" in previous reports.

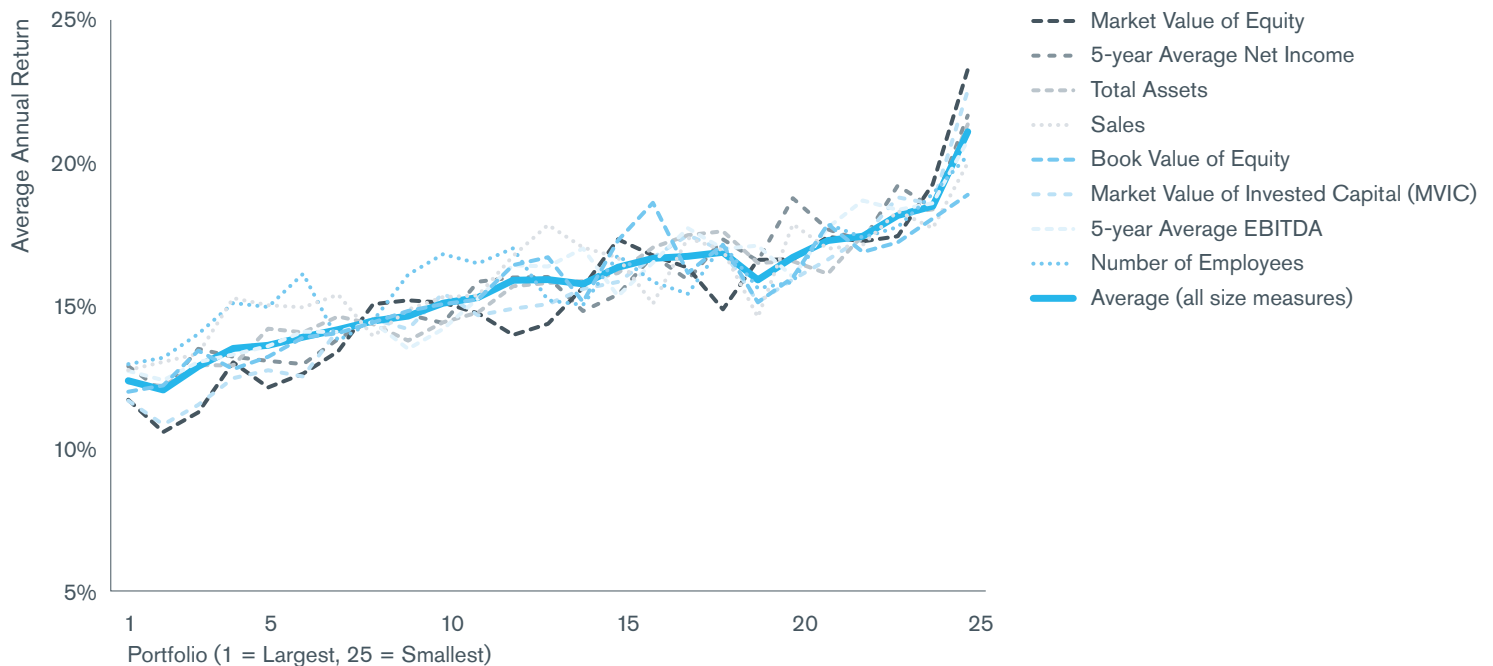
# The Size Study

The *Size Study* analyzes the relationship between stock returns and company size. In addition to presenting risk premia and size premia for 25 size-ranked portfolios using the traditional “market capitalization” measure, the *Size Study* also considers 7 other measures of company size, including book value of equity, 5-year average net income, market value of invested capital (MVIC), total assets, 5-year average EBITDA, sales, and number of employees.<sup>22</sup> As demonstrated in Graph 2, the data shows a clear inverse relationship between size and historical rates of return, regardless of how size is measured.

In Graph 2, as size *decreases* (from left to right), the average annual return over the study time horizon (1963–2010) tends to *increase* for each of the eight size measures.

For example, in the *2011 Report*, the average annual return of the portfolios made up of the largest companies (“Portfolio 1” for each of the eight size measures) was 12.4 percent, while the average annual return of the portfolios made up of the smallest companies (“Portfolio 25” for each of the eight size measures) was 21.1 percent.

**Graph 2: Average Annual Return, 8 Alternative Measures of Company Size 1963–2010**



<sup>22</sup> For a detailed discussion of portfolio creation methodology, see “Portfolio Methodology” on page 10.

# The Size Study

## What is Size?

The size of a company is one of the most important risk elements to consider when developing cost of equity estimates for use in valuing a firm. Traditionally, researchers have used market value of equity (i.e. “market capitalization”, or simply “market cap”) as a measure of size in conducting historical rate of return research. For instance, this is the basis of the small stock return series published in the *Stocks, Bonds, Bills and Inflation (SBB)* Valuation Yearbook.<sup>23</sup>

The *Size Study* measures the relationship between equity returns and up to eight alternative measures of size (including market capitalization). As illustrated in Graph 2, as size *decreases*, average annual return tends to *increase* for each of the eight size measures. Moreover, the “size effect” is not just evident for the smallest companies, but is evident for all but the largest groups of companies, including companies with a market capitalization in excess of several billions of dollars.

## Reasons for Using Alternative Measures of Size

There are various reasons for using alternative measures of size (in addition to market value of equity).

First, financial literature indicates a bias may be introduced when ranking companies by market value.<sup>24</sup> Second, a company's market capitalization may be affected by characteristics of the company other than size. In other words, some companies might be small because they are risky (high discount rate), rather than risky because they are small (low market capitalization). One simple example could be a company with a large asset base, but a small market capitalization as a result of high leverage or depressed earnings. Another example could be a company with large sales or operating income, but a small market capitalization due to being highly leveraged. Market capitalization may be an imperfect measure of the risk of a company's *operations*.

In addition, using alternative measures of size may have the practical benefit of removing the need to make a “guesstimate” of size for comparative purposes. Fundamental accounting measures (such as assets or net income) are generally readily available, while market capitalization, at least for a closely held firm, is not. This is related to another reason one might consider using alternative measures of size: potential “circularity” issues. When you are valuing a closely held company, you are trying to determine market capitalization. If you need to make a guesstimate of the subject company's market capitalization first in order to know which size premium to use, a “circularity” problem may be introduced.<sup>25</sup>

Finally, when doing analysis of any kind it is generally prudent to approach things from multiple directions if at all possible. This is good practice for several reasons, with the most important being that it has the potential of strengthening the conclusions of the analysis. For instance, we intuitively place more faith in a poll of 1,000 people than a poll of 10 people.<sup>26</sup>

<sup>23</sup> *Stocks, Bonds, Bills and Inflation Valuation Yearbook*, Morningstar (formerly Ibbotson Associates) (2011).

<sup>24</sup> “A Critique of Size Related Anomalies,” Jonathan Berk, *Review of Financial Studies*, vol. 8, no. 2 (1995).

<sup>25</sup> For a complete discussion of the history of the size premium and criticisms of the size premium, see chapter thirteen in *Cost of Capital: Applications and Examples* 3rd ed. by Shannon Pratt and Roger Grabowski, Wiley (2008); or chapter fourteen in *Cost of Capital: Applications and Examples* 4th ed. by Shannon Pratt and Roger Grabowski, Wiley (2010).

<sup>26</sup> The same logic can be extended to sources of valuation data. Both the *SBB* Yearbook and the *Duff & Phelps Risk Premium Report* are excellent sources of valuation data, and both are recommended for use in developing discount rates.

# The Size Study

While the A and B exhibits present different types of risk premia, both the A and B exhibits' 25 portfolios are ranked by the *same* eight alternative measures of size, which are described in Table 2.<sup>27</sup>

**Table 2: Eight Alternative Measures of Size**

<b>Exhibits A-1 and B-1</b> Market value of common equity (common stock price times number of common shares outstanding).	<b>Exhibits A-5 and B-5</b> Total Assets (as reported on the balance sheet).
<b>Exhibits A-2 and B-2</b> Book value of common equity (does not add back the deferred tax balance)	<b>Exhibits A-6 and B-6</b> 5-year average earnings before interest, income taxes, depreciation and amortization (EBITDA) for the previous five fiscal years (operating income before depreciation plus non-operating income).
<b>Exhibits A-3 and B-3</b> 5-year average net income for previous five fiscal years (net income before extraordinary items).	<b>Exhibits A-7 and B-7</b> Sales (net).
<b>Exhibits A-4 and B-4</b> Market value of invested capital (MVIC) (market value of common equity plus carrying value of preferred stock plus long-term debt (including current portion) and notes payable).	<b>Exhibits A-8 and B-8</b> Number of employees (number of employees, either at year-end or yearly average, including part-time and seasonal workers and employees of consolidated subsidiaries; excludes contract workers and unconsolidated subsidiaries).

Each of the exhibits A-1 through A-8 and B-1 through B-8 displays one line of data for each of the 25 size-ranked portfolios. The A and B exhibits include the statistics outlined in Table 3.

For comparative purposes, the average returns from the *S&B* series for Large Companies (essentially the S&P 500 Index), Small Companies, and Long-Term Government Bond Income Returns for the period 1963 through the latest year are also reported in each exhibit.<sup>28</sup>

**Table 3: Statistics Reported for 25 size-ranked portfolios in the Size Study's A and B Exhibits**

## Exhibits A-1 through A-8

- Average of the sorting criteria (e.g., average number of employees) for the latest year used in determining the size of the companies (i.e., the size criteria when the latest year's portfolios are formed). For example, the market value in exhibit A-1 is the market value of equity at the beginning of the latest year. The other size criteria are based on what was known at the beginning of the latest year when the portfolios are formed.

- The number of companies in each portfolio at the beginning of the latest year.

- Beta calculated using the "sum beta" method applied to monthly returns for 1963 through the latest year (see the *2011 S&B Valuation Yearbook* pp. 77-78 for a description of the "sum beta" method).

- Standard deviation of annual historical equity returns.

- Geometric average historical equity return since 1963.

- Arithmetic average historical equity return since 1963.

- Arithmetic average historical risk premium over long-term Treasuries (average return on equity in excess of long-term Treasury bonds) since 1963 ( $RP_{m+s}$ ).

- "Smoothed" average historical risk premium: the fitted premium from a regression with the average historical risk premium as dependent variable and the logarithm of the average sorting criteria as independent variable. (We present the coefficients and other statistics from this regression analysis in the top right hand corner of the exhibits) ( $RP_{m+s}$ ).

- Average carrying value of preferred stock plus long-term debt (including current portion) plus notes payable ("Debt") as a percent of MVIC since 1963.

## Exhibits B-1 through B-8

- Average of the sorting criteria (e.g., average number of employees) for the latest year used in determining the size of the companies (i.e., the size criteria when the latest year's portfolios are formed). For example, the market value in exhibit B-1 is the market value of equity at the beginning of the latest year. The other size criteria are based on what was known at the beginning of the latest year when the portfolios are formed.

- Beta estimate calculated using the "sum beta" method applied to monthly returns for 1963 through the latest year (see the *2011 S&B Valuation Yearbook* pp. 77-78 for a description of the "sum beta" method).

- Arithmetic average historical equity return since 1963.

- Arithmetic average historical risk premium over long-term Treasuries (average return on equity in excess of long-term Treasury bonds) since 1963 ( $RP_{m+s}$ ).

- Indicated CAPM premium, calculated as the beta of the portfolio multiplied by the average historical market risk premium since 1963 (measured as the difference between *S&B* Large Stock total returns and *S&B* income returns on long-term Treasury bonds).

- Premium over CAPM, calculated by subtracting the "Indicated CAPM Premium" from the "Arithmetic Risk Premium" ( $RP_s$ ).

- "Smoothed" Premium over CAPM: the fitted premium from a regression with the historical "Premium over CAPM" as dependent variable and the logarithm of the average sorting criteria as independent variable ( $RP_s$ ).

<sup>27</sup> For a detailed description of the Standard and Poor's *Compustat* data items used in the *Risk Premium Report*, please see Appendix A (in the full *Report*).

<sup>28</sup> Source: Morningstar *EnCorr Analyzer* software.

# The Size Study

## The Difference between the A Exhibits and the B Exhibits

The results of the *Size Study* are presented in Exhibits A-1 through A-8 and Exhibits B-1 through B-8. The main difference between the A and B exhibits is how they are used: the A exhibits are used if you are using a “buildup” method to develop cost of equity capital estimates, and the B exhibits are used if you are using the capital asset pricing model (CAPM) to develop cost of equity capital estimates. This difference in usage is a function of the type of “risk premia” presented in each of the exhibits:

- The A exhibits provide risk premia over the risk free rate in terms of the total effect of *market risk* and *size risk* for 25 portfolios ranked by eight alternative measures of size ( $RP_{m+s}$ ). These premia can be added to a risk free rate ( $R_f$ ) to estimate cost of equity capital (COE) in a “buildup” model.
- The B exhibits provide risk premia over CAPM (“size premia”) in terms of *size risk* for 25 portfolios ranked by eight alternative measures of size ( $RP_s$ ). These premia are commonly known as “beta-adjusted size premia”, or simply “size premia”. These premia can be added as a size adjustment to a basic CAPM to estimate cost of equity capital (COE).<sup>29</sup>

<sup>29</sup> The basic CAPM formula is  $COE = Risk\ Free\ Rate + (Beta \times ERP)$ . A “modified CAPM” refers to the common modification to the CAPM formula that is used to incorporate an adjustment for size:  $COE = Risk\ Free\ Rate + (Beta \times ERP) + Size\ Premium$ . Please note that the modified CAPM as presented is *after* addition of a size premium and *prior* to the addition of any company-specific risk premiums that may be applicable.

# The Size Study

## The Difference Between “Risk Premium Over the Risk Free Rate” and “Risk Premium Over CAPM”

The *Size Study* measures the relationship between equity returns and up to eight measures of size, including market capitalization. As size decreases, returns tend to increase.

The *Size Study* develops two primary types of risk premia, those that can be added to a risk free rate if you are using the buildup method (found in Exhibits A-1 through A-8), and premia over CAPM, which are commonly referred to as “beta adjusted size premia”, or simply “size premia” (found in Exhibits B-1 through B-8). Size premia can be added as a size adjustment if you are using the capital asset pricing model (CAPM).

### Risk Premium Over Risk Free Rate, $RP_{m+s}$

“Risk premium over the risk free rate” represent the difference between the historical (observed) return of equities over the return of the risk free rate. A long-run average historical risk premium is often used as an indicator of the expected risk premium of a typical equity investor. Returns are based on dividend income plus capital appreciation and represent returns after corporate taxes (but before owner-level taxes).

To estimate historical risk premiums, the average rate of return for each of the 25 size-based portfolios is calculated over the sample period, and then the average income return of long-term Treasury bonds (using *S&P* data) over the same period is subtracted.

The result is a clear negative relationship between size and premium over long-term bond yields (i.e. as size decreases, the return over the risk free rate increases). This difference is a measure of risk in terms of the total effect of *market risk* and *size risk*.

In Figure 5, for example, an abbreviated version of Exhibit A-6 is shown. The average annual arithmetic return for Portfolio 25 is 20.79 percent over the time period 1963–2010, and the average annual long-term Treasury income return over this period was 6.90%. This implies *actual* excess returns of 13.89 percent (20.79% - 6.90%) for this portfolio.

Because these premia have an embedded measure of market (i.e. “*beta*”) risk, these premia are appropriate for use in “buildup” methods that do not already include a measure of market risk, but are *not* appropriate for use in models (e.g. CAPM) that already have a measure of market risk (in the form of *beta*).

- Risk premia over the risk free rate ( $RP_{m+s}$ ) are presented in Exhibits A-1 through A-8. In the *2011 Report*, these risk premia are calculated over the period 1963 (the year that the *Compustat* database was inaugurated) through December 2010.

**Figure 5: Calculating Risk Premia Over the Risk Free Rate ( $RP_{m+s}$ )**

### Companies Ranked by 5-Year Average EBITDA

Historical Equity Risk Premium: Average Since 1963

Data for Year Ending December 31, 2010

#### Exhibit A-6

Portfolio Rank by Size	Average EBITDA (\$mils.)	Log of Average EBITDA	Number as of 2010	Beta (SumBeta) Since '63	Standard Deviation of Returns	Geometric Average Return	Arithmetic Average Return	Arithmetic Average Risk Premium	Smoothed Average Risk Premium	Average Debt/MVIC
1	17,770	4.25	33	0.79	16.26%	11.46%	12.71%	5.81%	4.01%	23.33%
2	4,983	3.70	32	0.84	16.02%	11.14%	12.39%	5.48%	5.66%	29.11%
←					///	→				
25	14	1.15	417	1.31	28.71%	17.15%	20.79%	13.89%	13.28%	22.86%
Large Stocks (Ibbotson <i>S&amp;P</i> data)						9.84%	11.30%	4.40%		
Small Stocks (Ibbotson <i>S&amp;P</i> data)						13.72%	16.53%	9.63%		
Long-Term Treasury Income (Ibbotson <i>S&amp;P</i> data)						6.88%	6.90%			

# The Size Study

## Risk Premium Over CAPM ("Size Premium"), $RP_s$

"Risk Premium over CAPM" represent the difference between historical (observed) excess return and the excess return predicted by CAPM. The historical excess return of portfolios comprised of smaller companies is greater than the excess return predicted by the CAPM for these portfolios. The difference is a measure of risk in terms of the effect of *size risk*, and is commonly referred to as a "beta-adjusted size premium", or simply "size premium".

For each portfolio, a size premium is calculated using the methodology for doing so as described in the *SBBI Valuation Yearbook*.<sup>30</sup> The formula for this adjustment is:

$$\text{Size Premium} = \text{Portfolio Premium} - (\text{Portfolio Beta} \times \text{Realized Market Premium})$$

where:

**Size premium:** the difference in historical excess returns (i.e. what "actually happened"), and the excess returns predicted by CAPM.

**Portfolio premium:** the actual return over the risk-free interest rate (i.e. "excess return") earned by a given portfolio between 1963 and 2010.

**Portfolio beta:** the beta estimated relative to the S&P 500 Index using annual returns between 1963 and 2010.

**Realized market premium:** the average annual excess return of the S&P 500 Index between 1963 and 2010 over the long-term risk free rate.

This adjustment can be thought of as simply "*what actually happened*" (the portfolio premium) minus "*what CAPM predicted would happen*" (the portfolio beta times the realized market premium).<sup>31</sup>

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<sup>30</sup> 2011 Ibbotson *SBBI Valuation Yearbook* (Chicago, Morningstar, 2011), Chapter 7, "Firm Size and Return", pages 85–105.

<sup>31</sup> The basic CAPM equation is  $COE = R_f + (\beta \times ERP)$ , which can be rewritten as  $COE - R_f = (\beta \times ERP)$ . COE (i.e. "expected return") minus the risk free rate ( $R_f$ ) is, by definition, the "expected return over the risk free rate", and therefore, so is  $(\beta \times ERP)$ .

# The Size Study

For example, an abbreviated version of Exhibit B-6 is shown in Figure 6. The average annual arithmetic return for Portfolio 25 is 20.79 percent over the time period 1963–2010, and the average annual long-term Treasury income return over this period was 6.90 percent. This implies actual excess returns of 13.89 percent (20.79% – 6.90%) for this portfolio.

Portfolio 25 has a calculated beta<sup>32</sup> of 1.31, and the realized market premium over the 1963–2010 period is 4.40 percent<sup>33</sup>. This implies that predicted excess return according to CAPM is 5.76 percent (1.31 x 4.40%).

The size premium for Portfolio 25 in Exhibit B-6 is therefore 8.13 percent, which is “*what actually happened*” (13.89%) minus “*what CAPM predicted*” (5.76%).

The risk premia over CAPM (i.e. “size premia”) published in the *Risk Premium Report* are adjusted for beta. In other words, the portion of excess return that is not attributable to beta is controlled for, or removed, leaving only the size effect’s contribution to excess return. These premia are appropriate for use in the capital asset pricing model (CAPM), and in buildup methods that do not otherwise already have a measure of size risk.<sup>34</sup>

- Risk premia over CAPM, or “size premia” ( $RP_s$ ) are presented in Exhibits B-1 through B-8. In the *2011 Report*, these risk premia are calculated over the period 1963 (the year that the *Compustat* database was inaugurated) through December 2010.

**Figure 6: Calculating Size Premia ( $RP_s$ )**

**Companies Ranked by 5-Year Average EBITDA**  
 Historical Equity Risk Premium: Average Since 1963  
 Data for Year Ending December 31, 2010  
**Exhibit B-6**

Portfolio Rank by Size	Average EBITDA (\$mils.)	Log of Size	Beta (SumBeta) Since '63	Arithmetic Average Return	Arithmetic Average Risk Premium	Indicated CAPM Premium	Premium over CAPM	Smoothed Premium over CAPM
1	17,770	4.25	0.79	12.71%	5.81%	3.45%	2.36%	0.65%
2	4,983	3.70	0.84	12.39%	5.48%	3.68%	1.81%	1.80%
⬅				➡				
				///				
25	14	1.15	1.31	20.79%	13.89%	5.76%	8.13%	7.14%
Large Stocks (Ibbotson <i>S&amp;B</i> I data)				11.30%	4.40%			
Small Stocks (Ibbotson <i>S&amp;B</i> I data)				16.53%	9.63%			
Long-Term Treasury Income (Ibbotson <i>S&amp;B</i> I data)				6.90%				

<sup>32</sup> The betas presented in the *Risk Premium Report* are “sum” betas. Smaller companies generally trade more infrequently and exhibit more of a “lagged” price reaction (relative to the market) than do large stocks. One of the ways of capturing this lag movement is called sum beta. See Ibbotson, Roger G., Paul D. Kaplan, and James D. Pearson. “Estimates of Small-Stock Betas Are Much Too Low,” *Journal of Portfolio Management*, Summer 1997.

<sup>33</sup> As derived from the average difference in the annual average returns of the S&P 500 Index and *S&P* long-term government Treasury bond income returns. Source: Morningstar *EnCorr Analyzer* software.

<sup>34</sup> For example, the size premia presented in Exhibit B cannot be used in “Buildup 1”. The Buildup 1 method uses “risk premia over the risk free rate” (from Exhibit A) that already have a measure of risk in terms of the total effect of market risk and size risk, ( $RP_{m+s}$ ). Using size premia in Buildup 1 would be “double counting” size risk.



# The Size Study

## Overview of Methods Used to Estimate Cost of Equity Capital Using the Size Study

The *Size Study* provides two methods of estimating COE for a subject company, Buildup 1 and CAPM, plus one method for estimating unlevered COE (the cost of equity capital assuming a firm is financed 100% with equity and 0% debt).<sup>35</sup>

Some users of the *Report* have inquired whether the *Size Study* can be used in conjunction with the industry risk premia (IRPs) published in the *SBBJ Valuation Edition Yearbook*, so we also include an alternative method in which a rudimentary adjustment is made to an IRP and then utilized in a modified buildup model, Buildup 2, that includes a separate variable for the industry risk premium.<sup>36</sup> These methods are summarized below in equation format, and summarized in Figure 7 in graphical “building blocks” format.

### 1) Buildup 1

$$COE_{Buildup\ 1} = (Risk\ Free\ Rate) + (Risk\ Premium\ in\ Excess\ of\ (over)\ the\ Risk\ Free\ Rate) + (Equity\ Risk\ Premium\ Adjustment)$$

Example 1a: using guideline portfolios: page 31 (in the full *Report*).

Example 1b: using regression equations: page 33 (in the full *Report*).

### 2) Buildup 1-Unlevered

$$COE_{Buildup\ 1-Unlevered} = (Risk\ Free\ Rate) + (Unlevered\ Risk\ Premium\ in\ Excess\ of\ (over)\ the\ Risk\ Free\ Rate) + (Equity\ Risk\ Premium\ Adjustment)$$

Example 2a: using guideline portfolios: page 40 (in the full *Report*).

Example 2b: using regression equations: page 44 (in the full *Report*).

### 3) Capital asset pricing model (CAPM)

$$COE_{CAPM} = (Risk\ Free\ Rate) + (Beta \times Equity\ Risk\ Premium) + (Size\ Premium)$$

Example 3a: using guideline portfolios: page 29

Example 3b: using regression equations: page 32

### 4) Buildup 2

$$COE_{Buildup\ 2} = (Risk\ Free\ Rate) + (Equity\ Risk\ Premium) + (Size\ Premium) + (Adjusted\ Industry\ Risk\ Premium)$$

Example 4a: using guideline portfolios: page 57 (in the full *Report*).

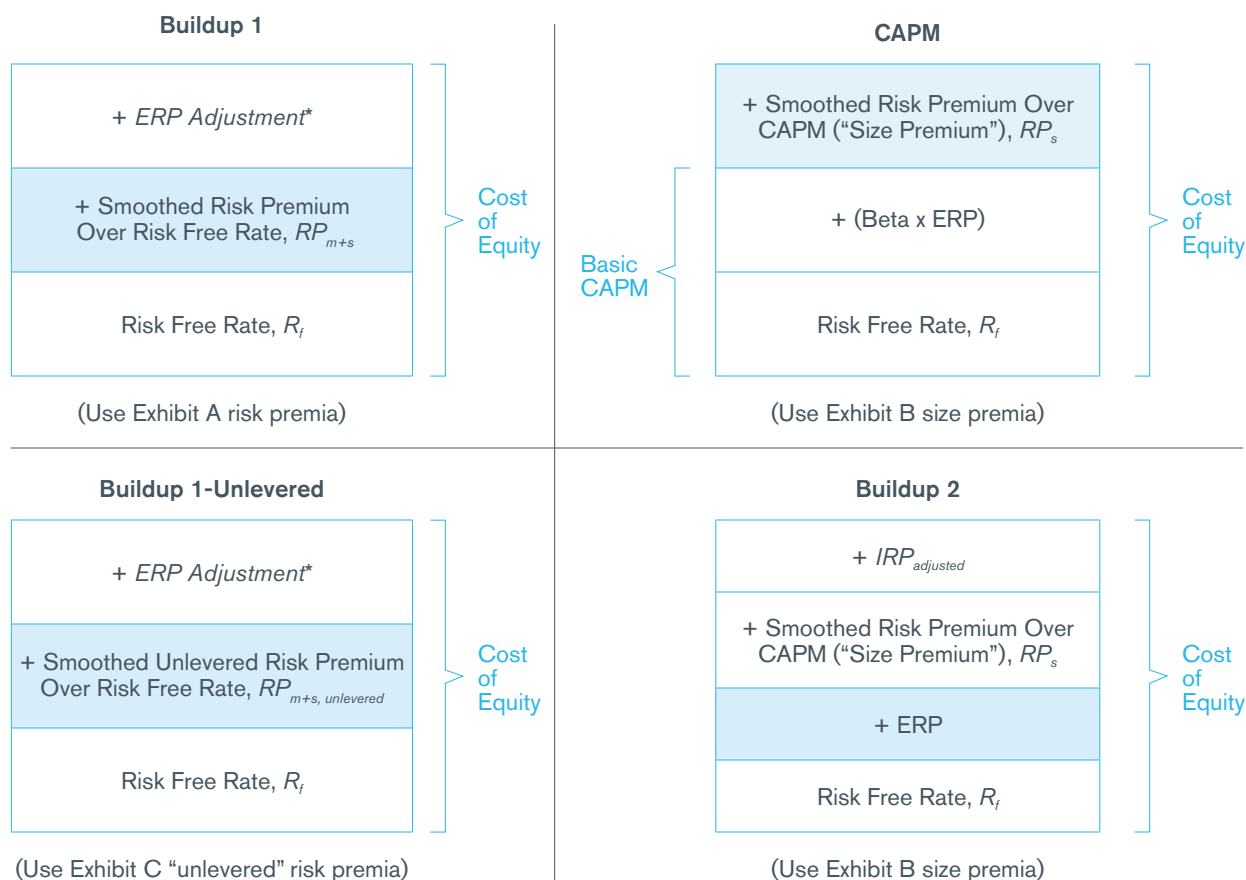
Example 4b: using regression equations: page 58 (in the full *Report*).

<sup>35</sup> Unlevered risk premia over the risk free rate are presented in Exhibits C-1 through C-8 (in the full *Report*).

<sup>36</sup> Duff & Phelps does not publish IRPs. A source of IRPs is Morningstar's *Ibbotson SBBJ Valuation Yearbook*, Table 3-5.

# The Size Study

Figure 7: Four Methods of Estimating Cost of Equity Capital with the *Size Study*<sup>37</sup>



\* ERP Adjustment: The difference between the historical (1963–2010) equity risk premium (ERP) and a user of the 2011 Report's own forward ERP estimate:

$$ERP \text{ Adjustment} = \text{User's ERP} - \text{Historical ERP (1963–2010)}$$

The ERP Adjustment is made only in the "Buildup 1", "Buildup 1-Unlevered", "Buildup 1-High-Financial-Risk", "Buildup 3", and "Buildup 3-Unlevered" methods. Please refer to the individual examples provided for these models for more information.

NOTE: This section includes an example of using the Report's size premia data to estimate cost of equity capital using the CAPM method, plus an overview of the unlevering/relevering methodology employed in the 2011 Report.

Complete examples for using the Report's size premia and risk premia to estimate cost of equity capital using the "Buildup 1", "Buildup 1-Unlevered", and "Buildup 2" methods are available in the full version of the 2011 Report.

<sup>37</sup> The relative size of the "building blocks" in Figure 7 do not necessarily represent the relative size of the various inputs.

# The Size Study

As shown in Figure 8, there are up to eight alternative size measures that can be used with any of the four methods of estimating COE provided by the *Size Study*. It is important to note that it would not be unusual for fewer than eight of these measures to be available for any given subject company. For example, market value of equity will probably not be available for a closely-held company, nor will market value of invested capital (in which market value of equity is embedded). In cases where fewer than eight size measures are available, it is generally acceptable to use the size measures that are available.

**Figure 8: Subject Company Size Characteristics**  
(used in all examples)

	Size Measure (in \$millions)	Appropriate Exhibit			
		Buildup 1	Buildup 1 - Unlevered	CAPM	Buildup 2
Market Value of Equity	\$120	A-1	C-1	B-1	B-1
Book Value of Equity	\$100	A-2	C-2	B-2	B-2
5-year Average Net Income	\$10	A-3	C-3	B-3	B-3
Market Value of Invested Capital	\$180	A-4	C-4	B-4	B-4
Total Assets	\$300	A-5	C-5	B-5	B-5
5-year Average EBITDA	\$30	A-6	C-6	B-6	B-6
Sales	\$250	A-7	C-7	B-7	B-7
Number of Employees	200	A-8	C-8	B-8	B-8

Figure 8 also includes the data exhibits in which the appropriate risk premia for each of the size measures can be found. For example, for use in the Buildup 1 method, risk premia over the risk free rate ( $RP_{m+s}$ ) for "Total Assets" are found in Exhibit A-5. For use in the CAPM method, the appropriate premia over CAPM ( $RP_{st}$ , or "size premia") for "Total Assets" are found in Exhibit B-5.

The "C" exhibits, which have not been discussed in detail thus far, provide unlevered versions of the risk premia over the risk free rate found in the A Exhibits. These unlevered premia ( $RP_{m+s, unlevered}$ ) can be used to estimate cost of equity capital assuming a firm is financed 100% with equity and 0% debt.<sup>38</sup>

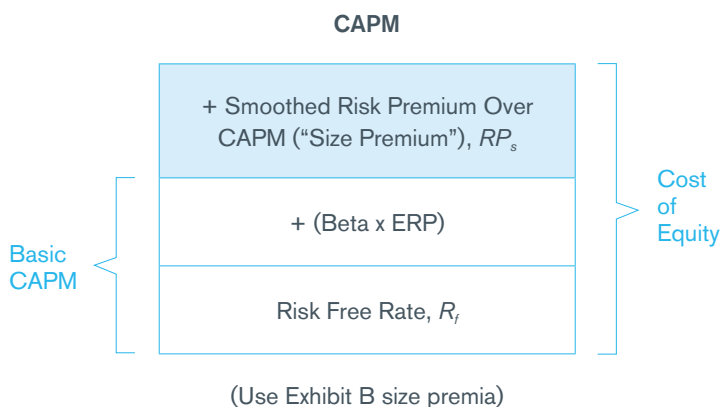
The C exhibits also provide another important function—they serve as a "link" between the *Size Study* and the *Risk Study*. These exhibits can be used to compare a subject company's fundamental risk characteristics to the fundamental risk characteristics of portfolios made up of similarly-sized companies. Examples of this important functionality are found in the *Risk Study*, which begins on page 39.

In each of the following examples of using the *Size Study* to estimate COE, the subject company size measures summarized in Figure 8 will be used (total assets of \$300 million, for instance, will be used in all examples). Also, the long-term risk free rate, ERP, and the *ERP Adjustment* established in the first example (Example 1a, Buildup 1 using "guideline portfolios") will be used (as appropriate) for all the subsequent examples, mirroring the fact that for any given valuation engagement, the same risk free rate and ERP will generally be used in each of the models presented by the individual analyst. Please note that for any given valuation engagement these inputs may (and probably will) be different than the ones used in the examples.

<sup>38</sup> The D exhibits also include "unlevered" risk premia, but these are unlevered versions of the corresponding "levered" risk premia found in the *Risk Study*'s D exhibits. The unlevered premia in the C exhibits are unlevered versions of the "levered" risk premia found in the *Size Study*'s A exhibits.

# The Size Study

## Estimating Cost of Equity Capital Using the "CAPM" Method



The capital asset pricing model (CAPM) is the most widely used method for estimating the cost of equity capital. For example, one survey found that while many firms use multiple methods of estimating the cost of equity capital, 75% of them use the CAPM.<sup>70</sup> Despite its criticisms, the CAPM has been one of the most widely used models for estimating the cost of equity capital for more than 30 years. The basic CAPM formula for estimating the cost of equity capital (COE) is:

$$COE_{CAPM} = R_f + (\beta \times ERP)$$

where:

- $R_f$  = the risk free rate as of the valuation date (typically a long-term US Treasury bond yield)
- $\beta$  = a measure of market (called systematic) risk of a stock; the sensitivity of changes in the returns (dividends plus price changes) of a stock relative to changes in the returns of a specific market benchmark or index.<sup>71</sup>
- $ERP$  = the equity risk premium. The ERP is the rate of return added to a risk free rate to reflect the additional risk of equity instruments over risk free instruments.

Research tells us that the CAPM often misprices risk for certain investments. Specifically, researchers have observed that commonly used methods of measuring risk used in the CAPM (specifically, beta) often understate the risk (and thus understate the required return) for small company stocks. Examination of market evidence shows that within the context of CAPM, beta does not fully explain the difference between small company returns and large company returns. In other words, the historical (observed) excess return of portfolios comprised of smaller companies is greater than the excess return predicted by the CAPM for these portfolios. This "premium over CAPM" is commonly known as a "beta-adjusted size premium" or simply "size premium".<sup>72</sup>

It follows that the size premium is a necessary correction to the basic CAPM because risk, as measured by the betas of smaller companies (even sum betas), is systematically underestimated.<sup>73</sup> Moreover, the size effect is not just evident for the smallest companies in the marketplace, but is evident for all but the largest groups of companies, including companies with a market capitalization in excess of \$1 billion. A common practice is to incorporate this evidence by adding a size premium to the CAPM formula when valuing companies that are comparatively small. The modified CAPM formula is<sup>74</sup>:

$$COE_{CAPM} = R_f + (\beta \times ERP) + RP_s$$

where:

$RP_s$  = the "beta-adjusted" size premium.

It is important to note that the risk premia over CAPM (i.e. "size premia") published in the *Risk Premium Report* are adjusted for beta.<sup>75</sup> In other words, the portion of excess return that is not attributable to beta is controlled for, or removed, leaving only the size effect's contribution to excess return. These premia are appropriate for use in the capital asset pricing model (CAPM), and in buildup methods that do not otherwise *already* have a measure of size risk.<sup>76</sup>

<sup>70</sup> John R. Graham and Campbell R. Harvey, "The Theory and Practice of Corporate Finance," *Journal of Financial Economics* (May 2001): 187–243.

<sup>71</sup> For the purposes of this report, the "market" is defined as the S&P 500 Index. The S&P 500 Index is a broad-based, market-capitalization-weighted index widely regarded as being representative of the overall market.

<sup>72</sup> A sample of academic research articles include: Rolf Banz, "The Relationship Between Return and Market Value of Common Stocks," *Journal of Financial Economics* (March 1981): 3–18; Eugene Fama and Kenneth French, "The Cross Section of Expected Stock Returns," *Journal of Finance* (June 1992): 427–486; Kent Daniel and Sheridan Titman, "Evidence on the Characteristics of Cross Sectional Variation in Stock Returns," *Journal of Finance* (March 1997): 1–33.

<sup>73</sup> The betas presented in the *Risk Premium Report* are "sum" betas. Smaller companies generally trade more infrequently and exhibit more of a "lagged" price reaction (relative to the market) than do large stocks. One of the ways of capturing this lag movement is called sum beta. See Ibbotson, Roger G., Paul D. Kaplan, and James D. Pearson, "Estimates of Small-Stock Betas Are Much Too Low," *Journal of Portfolio Management*, Summer 1997.

<sup>74</sup> A "modified CAPM" typically refers to the common modification to the CAPM formula that is used to incorporate an adjustment for size.

<sup>75</sup> For a detailed discussion of how premia over CAPM ("size premia") are calculated, see "The Difference Between 'Risk Premia Over the Risk Free Rate' and 'Risk Premia Over CAPM'" on page 22.

<sup>76</sup> For example, the size premia presented in Exhibit B cannot be used in "Buildup 1". The Buildup 1 method uses "risk premia over the risk free rate" (from Exhibit A) that *already* have a measure of risk in terms of the total effect of market risk and size risk, ( $RP_{m+s}$ ). Using size premia in Buildup 1 would be "double counting" size risk.

# The Size Study

Please note that base estimates of COE developed with the modified CAPM equation presented above are *after* addition of a size premium, but *prior* to the addition of any company-specific risk premiums ( $RP_u$ ) that the individual analyst may deem to be applicable.<sup>77</sup> Company-specific risk can be added by the individual analyst to the modified CAPM in the following fashion:

$$COE_{CAPM} = R_f + (\beta \times ERP) + RP_s + RP_u$$

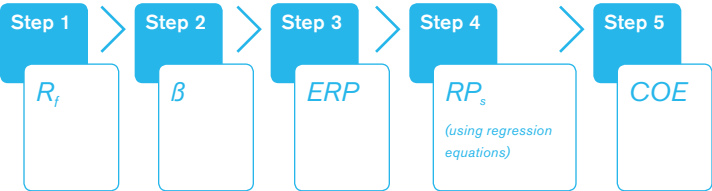
The *Risk Premium Report* provides two ways for analysts to match their subject company’s size (or risk) characteristics with the appropriate smoothed premia from the data exhibits: the “guideline portfolio” method and the “regression equation” method.<sup>78</sup> In general, the regression equation method is preferred because this method allows for interpolation between the individual guideline portfolios, although the guideline portfolio method is less complicated, and more direct.

Examples of both the guideline portfolio method and the regression equation method follow, starting with the simpler guideline portfolio method.

### Example 3a: CAPM Method (using guideline portfolios)

Four pieces of information are needed to estimate the cost of equity capital using the CAPM method and “guideline portfolios”: a risk free rate ( $R_f$ ), a beta ( $\beta$ ), an equity risk premium (ERP), and a risk premium over CAPM ( $RP_s$ , otherwise known as a beta-adjusted “size premium”). All of the information needed is summarized in Figure 27.

Figure 27: Information Needed to Estimate COE Using CAPM and Guideline Portfolios



This example utilizes the risk free rate ( $R_f$ ) and ERP that were established in Example 1a (see page 31 in the full *Report*). This mirrors the fact that for any given valuation engagement the same risk free rate and ERP will generally be used in each of the models presented by the individual analyst. For any given valuation engagement these inputs may (and probably will) be different than the ones used in the examples.

<sup>77</sup> The “u” in  $RP_u$  stands for nique risk or company-specific risk, and is also commonly referred to as nsystematic risk.  
<sup>78</sup> See pages 14–16 for a detailed explanation of the differences between the guideline portfolio method and the regression equation method.

# The Size Study

**Step 1, Risk Free Rate ( $R_f$ ):** The risk free rate is typically a long-term US Treasury bond yield as of the valuation date. This example utilizes the assumed long-term treasury yield of 4.1 percent established in Example 1a (page 31 in the full *Report*).

**Step 2, Beta ( $\beta$ ):** Beta is a measure of the sensitivity of changes in the returns (dividends plus price changes) of a stock relative to changes in returns of a specific market benchmark or index. Duff & Phelps does not currently publish company betas or peer group betas.<sup>79</sup> Because the sum betas calculated for the 25 size-ranked portfolios in the B exhibits are betas for a particular *size* of company (rather than a particular industry), they would in all likelihood *not* be appropriate for use within a CAPM estimate of COE, where the beta should be a measure of *market*, (or industry) risk. For this example, a beta of 1.2 is assumed.

**Step 3, Equity Risk Premium (ERP):** The ERP is the rate of return added to a risk free rate to reflect the additional risk of equity instruments over risk-free instruments. For this example, the *Duff & Phelps Recommended ERP* as of the end of 2010 (5.5%) is assumed.<sup>80, 81</sup>

**Step 4, Risk Premium Over CAPM (“size premium”) ( $RP_s$ ):** Match the various size measures of the subject company with the guideline portfolios composed of companies of similar size in Exhibits B-1 through B-8, and identify the corresponding smoothed average risk premium over CAPM (i.e. “size premium”).

The subject company in this example has a market value of equity of \$120 million, and the appropriate data exhibit is Exhibit B-1 (see Figure 8 on page 27). An abbreviated version of Exhibit B-1 is shown in Figure 28. Of the 25 portfolios, the portfolio that has an average market value *closest* to the subject company’s \$120 million market value is portfolio 25 (\$68 million). The corresponding smoothed average size premium is 8.02 percent (8.0 percent, rounded).

Match each of the subject company’s size measures in this fashion. For example, the second size measure for the subject company in this example is “book value of equity” of \$100 million. Of the 25 guideline portfolios in Exhibit B-2 (not shown here), the portfolio that has an average book value of equity *closest* to the subject company’s \$100 million book value is portfolio 25 (\$60 million). The corresponding smoothed average size premium is therefore 6.4 percent. After all of the available size measures for the subject company have been matched to the closest guideline portfolio in the appropriate exhibit and the corresponding smoothed average size premium has been identified for each, Step 4 is complete.

**Figure 28: Exhibit B-1 (abbreviated)**

## Companies Ranked by Market Value of Equity

Historical Equity Risk Premium: Average Since 1963

Data for Year Ending December 31, 2010

Portfolio Rank by Size	Average Mkt Value (\$mils.)	Log of Size	Beta (SumBeta) Since '63	Arithmetic Average Return	Arithmetic Average Risk Premium	Indicated CAPM Premium	Premium over CAPM	Smoothed Premium over CAPM
1	109,765	5.04	0.84	11.70%	4.80%	3.67%	1.13%	-1.22%
2	32,309	4.51	0.95	10.57%	3.67%	4.18%	-0.51%	0.31%
3	22,008	4.34	0.93	11.26%	4.35%	4.09%	0.27%	0.79%
///								
24	232	2.36	1.25	19.26%	12.36%	5.50%	6.86%	6.48%
25	68	1.83	1.29	23.28%	16.37%	5.67%	10.71%	<b>8.02%</b>

<sup>79</sup> Company betas and industry betas are available from multiple sources, including Bloomberg, MSCI, and Value Line.

<sup>80</sup> For more information on the equity risk premium, see *Cost of Capital: Applications and Examples* 4th ed., by Shannon P. Pratt and Roger J. Grabowski (John Wiley & Sons, Inc., 2010), Chapter 9, “Equity Risk Premium”, pages 115–158.

<sup>81</sup> For more information on cost of capital issues, including developing risk-free rates and ERP during periods of flight to quality, please visit [www.duffandphelps.com/costofcapital](http://www.duffandphelps.com/costofcapital).

# The Size Study

**Step 5, Estimate Cost of Equity (COE):** With the completion of Steps 1 through 4, the information needed to estimate a base cost of equity capital using the CAPM is now completed. The risk premia over CAPM ( $RP_s$ , or “size premia”) can now be added to the basic CAPM equation ( $COE_{CAPM} = R_f + (\beta \times ERP) + RP_s$ ) to estimate an indicated cost of equity capital (COE) for the subject company, as illustrated in Figure 29.

The range of COE estimates for the hypothetical subject company in this example is 16.5 percent to 18.7 percent, with an average of 17.3 percent, and a median of 17.0 percent. The mean represents the average estimate, but the mean can be unduly influenced by very large or very small “outliers”. For this reason, the median estimate is generally preferred to the mean. The median estimate tends to not be as heavily influenced by very large or very small outliers, and can be considered a measure of the “typical” estimate in the group.

Remember that the full CAPM equation is:

$$COE_{CAPM} = R_f + (\beta \times ERP) + RP_s + RP_u$$

The base COE estimates derived in this example are therefore *prior* to the addition of any other company-specific risk premiums ( $RP_u$ ) that the individual analyst may deem appropriate.

**Figure 29: CAPM COE Inputs (using guideline portfolios)**

	Size Measure (in \$millions)	Appropriate Exhibit	Guideline Portfolio	Step 1		Step 2		Step 3		Step 4		Step 5
				Risk Free Rate, $R_f$		Beta $\beta$		ERP		Smoothed Premium Over CAPM (size premium), $RP_s$		COE
Market Value of Equity	\$120	B-1	25	4.1%	+	(1.2)	x	5.5%)	+	8.0%	=	18.7%
Book Value of Equity	\$100	B-2	25	4.1%	+	(1.2)	x	5.5%)	+	8.4%	=	17.1%
5-year Average Net Income	\$10	B-3	24	4.1%	+	(1.2)	x	5.5%)	+	8.3%	=	17.0%
Market Value of Invested Capital	\$180	B-4	25	4.1%	+	(1.2)	x	5.5%)	+	7.7%	=	16.4%
Total Assets	\$300	B-5	24	4.1%	+	(1.2)	x	5.5%)	+	6.9%	=	16.0%
5-year Average EBITDA	\$30	B-6	24	4.1%	+	(1.2)	x	5.5%)	+	6.1%	=	16.0%
Sales	\$250	B-7	24	4.1%	+	(1.2)	x	5.5%)	+	5.8%	=	16.5%
Number of Employees	200	B-8	25	4.1%	+	(1.2)	x	5.5%)	+	6.0%	=	17.0%
Mean (average) values				4.1%	+	(1.2)	x	5.5%)	+	6.6%	=	17.3%
Median (typical) values				4.1%	+	(1.2)	x	5.5%)	+	6.3%	=	17.0%

NOTE: Some values intentionally blurred.

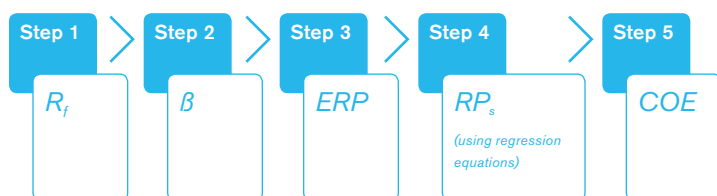
# The Size Study

## Example 3b: CAPM Method (using regression equations)

When the subject company size measures do not exactly match the respective average company size of the guideline portfolios, the data exhibits provide a straightforward way to interpolate an “exact” risk premium over CAPM between guideline portfolios using the “regression equation” method.

The *only* difference between estimating cost of equity capital (COE) using the CAPM method using “guideline portfolios” (as in the previous example) and estimating COE using the CAPM method using “regression equations” is how the risk premia over CAPM ( $RP_s$ , or “size premia”) are identified in Step 4.

**Figure 27: Information Needed to Estimate COE Using CAPM and Regression Equations**



In the previous example, the smoothed average risk premia over CAPM published in the *Report* for the appropriate guideline portfolios were used to estimate COE.<sup>82</sup> In this example, however, the regression equations found in each of the data exhibits will be used to calculate “custom” interpolated size premia, based upon the specific size measures of the subject company.

This example utilizes the long-term risk free rate ( $R_f$ ) and ERP established in a previous example (the *Size Study*’s Buildup 1 method using “guideline portfolios”; see page 31 in the full *Report*), and the Beta ( $\beta$ ), established for the previous example (Example 3a on page 29). This mirrors the fact that for any given valuation engagement the same inputs will generally be used in each of the models presented by the individual analyst. Please note that for any given valuation engagement these inputs may (and probably will) be different than the ones used in the examples. The only missing ingredients needed to estimate COE are the premia over CAPM, or size premia ( $RP_s$ ), as summarized in Figure 30.

**Figure 30: Needed—Smoothed Premia Over CAPM ( $RP_s$ , or “Size Premia”) Calculated Using Regression Equations**

			Step 1		Step 2		Step 3		Step 4		Step 5
	Size Measure (in \$millions)	Appropriate Exhibit	Risk Free Rate, $R_f$		Beta $\beta$		ERP		Smoothed Premium Over CAPM (size premium), $RP_s$		COE
Market Value of Equity	\$120	B-1	4.1%	+	(1.2)	x	5.5%	+	?	=	
Book Value of Equity	\$100	B-2	4.1%	+	(1.2)	x	5.5%	+	?	=	
5-year Average Net Income	\$10	B-3	4.1%	+	(1.2)	x	5.5%	+	?	=	
Market Value of Invested Capital	\$180	B-4	4.1%	+	(1.2)	x	5.5%	+	?	=	
Total Assets	\$300	B-5	4.1%	+	(1.2)	x	5.5%	+	?	=	
5-year Average EBITDA	\$30	B-6	4.1%	+	(1.2)	x	5.5%	+	?	=	
Sales	\$250	B-7	4.1%	+	(1.2)	x	5.5%	+	?	=	
Number of Employees	200	B-8	4.1%	+	(1.2)	x	5.5%	+	?	=	
Mean (average) values			4.1%	+	(1.2)	x	5.5%	+		=	
Median (typical) values			4.1%	+	(1.2)	x	5.5%	+		=	

<sup>82</sup> The smoothed risk premia published in the *Risk Premium Report* are based upon the average size (or risk) measure in each of the respective guideline portfolios.



# The Size Study

**Step 1, Risk Free Rate ( $R_f$ ):** The risk free rate is typically a long-term US Treasury bond yield as of the valuation date. This example utilizes the assumed long-term treasury yield of 4.1 percent used in Example 1a (page 31 in the full *Report*).

**Step 2, Beta ( $\beta$ ):** Beta is a measure of the sensitivity of a stock's price relative to movements of a specific market benchmark or index. For this example, the beta of 1.2 that was assumed in Example 3a (page 29) is assumed.

**Step 3, Equity Risk Premium (ERP):** The ERP is the rate of return added to a risk-free rate to reflect the additional risk of equity instruments over risk-free instruments. For this example, the *Duff & Phelps Recommended ERP* as of the end of 2010 (5.5%) is assumed.<sup>83, 84</sup>

**Step 4, Risk Premium Over CAPM ( $RP_s$ ):** The hypothetical subject company in this example has a market value of equity of \$120 million, and the appropriate *Size Study* data exhibit to use is Exhibit B-1<sup>85</sup>. In this case one would expect that the smoothed average premium over CAPM, or size premium, would fall somewhere between 6.48 percent (the smoothed size premium for Portfolio 24) and 8.02 percent (the smoothed size premium for Portfolio 25), as illustrated in Figure 31.

An easy way to calculate a custom interpolated risk premium over CAPM ( $RP_s$ , or "size premia") "in between" Portfolio 24 and Portfolio 25 is by using the regression equations provided for this purpose in each of the data exhibits. The regression equations are located in the same spot in each of the exhibits (see Figure 4 on page 16).<sup>86</sup>

**Figure 31: Exhibit B-1 (abbreviated)**

**Companies Ranked by Market Value of Equity**  
Historical Equity Risk Premium: Average Since 1963  
Data for Year Ending December 31, 2010

Portfolio Rank by Size	Average Mkt Value (\$mils.)	Log of Size	Beta (SumBeta) Since '63	Arithmetic Average Return	Arithmetic Average Risk Premium	Indicated CAPM Premium	Premium over CAPM	Smoothed Premium over CAPM ( $RP_s$ )
1	109,765	5.04	0.84	11.70%	4.80%	3.67%	1.13%	-1.22%
2	32,309	4.51	0.95	10.57%	3.67%	4.18%	-0.51%	0.31%
3	22,008	4.34	0.93	11.26%	4.35%	4.09%	0.27%	0.79%
← /// →								
24	232	2.36	1.25	19.26%	12.36%	5.50%	6.86%	6.48%
Subject Company	120	----->						?
25	68	1.83	1.29	23.28%	16.37%	5.67%	10.71%	8.02%

<sup>83</sup> For more information on the equity risk premium, see *Cost of Capital: Applications and Examples* 4th ed., by Shannon P. Pratt and Roger J. Grabowski (John Wiley & Sons, Inc., 2010), Chapter 9, "Equity Risk Premium", pages 115–158.

<sup>84</sup> For more information on cost of capital issues, including developing risk-free rates and ERP during periods of flight to quality, please visit [www.duffandphelps.com/costofcapital](http://www.duffandphelps.com/costofcapital).

<sup>85</sup> The same eight size measures (for a hypothetical subject company) are used in *all* examples of estimating COE using the *Size Study*, as outlined in Figure 8 on page 27.

<sup>86</sup> In addition to regression equations for interpolating risk premia between guideline portfolios in the *Size Study*'s A and B exhibits, the *Risk Study*'s D exhibits also provide regression equations for easy interpolation of risk premia between guideline portfolios, as do the C exhibits (for unlevered risk premia).

## The Size Study

The regression equation provided in Exhibit B-1, which includes 25 portfolios ranked by market value<sup>87</sup>, is:

$$\text{Smoothed Premium} = 13.285\% - 2.879\% * \text{Log}(\text{Market Value})$$

To calculate an interpolated smoothed risk premium over CAPM ( $RP_s$ , "size premia") for the subject company's \$120 million market value, substitute the market value into the regression equation as follows<sup>88</sup>:

$$\text{Smoothed Premium} = 13.285\% - 2.879\% * \text{Log}(120)$$

$$7.3\% = 13.285\% - 2.879\% * 2.08$$

Continue interpolating smoothed risk premium over CAPM for each size measure available for the subject company using the regression equations from the data exhibits. For example, the second size measure for the subject company is "book value of equity" of \$100 million. The equation found on Exhibit B-2 is:

$$\text{Smoothed Premium} = 9.849\% - 1.957\% * \text{Log}(\text{Book Value})$$

The interpolated smoothed risk premium over CAPM is therefore 5.9 percent ( $9.849\% - 1.957\% * 2$ ). After interpolating smoothed size premia for all of the subject company's available size measures, Step 4 is complete, as shown in Figure 32.

**Figure 32: Calculation of Risk Premia Over CAPM ( $RP_s$ ) Using Regression Equations**

Appropriate Exhibit	Size Measure	Subject Company Size Measures (in \$millions)	Appropriate Regression Equation	Step 4 Smoothed Risk Premium Over CAPM (size premium), $RP_s$
B-1	Market Value of Equity	120	Smoothed Premium = $13.285\% - 2.879\% * \text{Log}(\text{Market Value})$	= 7.3%
B-2	Book Value of Equity	100	Smoothed Premium = $9.849\% - 1.957\% * \text{Log}(\text{Book Value})$	= 5.9%
B-3	5-year Average Net Income	10	Smoothed Premium = $8.559\% - 2.119\% * \text{Log}(\text{Net Income})$	= 6.5%
B-4	Market Value of Invested Capital	180	Smoothed Premium = $12.879\% - 2.879\% * \text{Log}(\text{MVIC})$	= 7.0%
B-5	Total Assets	300	Smoothed Premium = $11.338\% - 2.125\% * \text{Log}(\text{Assets})$	= 6.1%
B-6	5-year Average EBITDA	30	Smoothed Premium = $9.539\% - 2.082\% * \text{Log}(\text{EBITDA})$	= 6.4%
B-7	Sales	250	Smoothed Premium = $10.104\% - 1.711\% * \text{Log}(\text{Sales})$	= 6.5%
B-8	Number of Employees	200	Smoothed Premium = $11.277\% - 1.832\% * \text{Log}(\text{Employees})$	= 7.1%

NOTE: Some values intentionally blurred.

<sup>87</sup> Figure 8 on page 27 lists the appropriate "B" exhibits in which the size premia for each of the eight size measures can be found.

<sup>88</sup> Please note that the logarithmic relationship is base-10, and that the financial size data is in millions of dollars, such that the log of \$10 million is log(10), not log(10,000,000). The formula to calculate a value's base-10 logarithm in Microsoft Excel is "=log(value)". The "\*" used in the regression equation is the symbol used in Microsoft Excel to denote the multiplication symbol, "x". The "\*" format is used to denote multiplication in the regression equations in the data exhibits.

# The Size Study

**Step 5, Estimate Cost of Equity (COE):** With the completion of Steps 1 through 4, the information needed to estimate a base cost of equity capital using the CAPM (using regression portfolios) is now completed. The risk premiums over CAPM ( $RP_s$ , or “size premia”) can now be added to the basic CAPM equation ( $COE_{CAPM} = R_f + (\beta \times ERP) + RP_s$ ) to estimate an indicated cost of equity capital (COE) for the subject company, as illustrated in Figure 33.

The range of COE estimates for the hypothetical subject company in this example is 16.6 percent to 18.0 percent, with an average of 17.2 percent, and a median of 17.2 percent. The mean estimate is the simple average of the COE estimates, but the mean can be unduly influenced by very large or very small “outliers”. For this reason, the median COE estimate is generally preferred to the mean. The median tends to not be as heavily influenced by very large or very small outliers, and can be considered a measure of the “typical” COE estimate in the group.

Remember that the full CAPM equation is:

$$COE_{CAPM} = R_f + (\beta \times ERP) + RP_s + RP_u$$

The base COE estimates derived in this example are therefore *prior* to the addition of any company-specific risk premiums ( $RP_u$ ) that the individual analyst may deem appropriate.

**Figure 33: CAPM COE Inputs (using regression equations)**

			Step 1		Step 2		Step 3		Step 4		Step 5
	Size Measure (in \$millions)	Appropriate Exhibit	Risk Free Rate, $R_f$		Beta $\beta$		ERP		Smoothed Premium Over CAPM (size premium), $RP_s$		COE
Market Value of Equity	\$120	B-1	4.1%	+	(1.2)	x	5.5%)	+	7.3%	=	18.0%
Book Value of Equity	\$100	B-2	4.1%	+	(1.2)	x	5.5%)	+	5.9%	=	16.6%
5-year Average Net Income	\$10	B-3	4.1%	+	(1.2)	x	5.5%)	+	6.5%	=	17.2%
Market Value of Invested Capital	\$180	B-4	4.1%	+	(1.2)	x	5.5%)	+	7.3%	=	17.7%
Total Assets	\$300	B-5	4.1%	+	(1.2)	x	5.5%)	+	6.1%	=	16.8%
5-year Average EBITDA	\$30	B-6	4.1%	+	(1.2)	x	5.5%)	+	6.4%	=	17.1%
Sales	\$250	B-7	4.1%	+	(1.2)	x	5.5%)	+	6.2%	=	16.7%
Number of Employees	200	B-8	4.1%	+	(1.2)	x	5.5%)	+	7.1%	=	17.8%
Mean (average) values			4.1%	+	(1.2)	x	5.5%)	+	6.5%	=	17.2%
Median (typical) values			4.1%	+	(1.2)	x	5.5%)	+	6.5%	=	17.2%

NOTE: Some values intentionally blurred.

# The Size Study

## Unlevered Cost of Equity Capital

In the *2011 Report*, the methodology and assumptions for unlevering risk premiums reported in Exhibits C-1 through C-8 were updated.<sup>55</sup> Unlevered premia are used to estimate cost of equity capital assuming a firm is financed 100% with equity and 0% debt. Generally, as the percentage of leverage (debt) in a company's capital structure increases, the cost of equity capital increases.

The unlevered realized risk premiums displayed in exhibits C-1 through C-8 are also informative in that they generally indicate that the market views smaller companies' operations to be riskier than the operations of larger companies (i.e., unlevered risk premiums increase as size decreases).

## Overview of the Current Methodology and Assumptions Used to Unlever Risk Premia in the 2011 Risk Premium Report

The average (levered) risk premia presented in Exhibits A-1 through A-8 are unlevered as follows<sup>56</sup>:

$$RP_{unlevered} = RP_{levered} - [(W_d / W_e) \times (\beta_u - \beta_d) \times RP_m]$$

where:

$RP_{unlevered}$  = Unlevered realized risk premium over the risk free rate

$RP_{levered}$  = Levered realized risk premium over the risk free rate

$\beta_u$  = Unlevered equity beta<sup>57</sup>

$\beta_d$  = Debt beta, assumed equal to 0.1

$RP_m$  = General equity risk premium (ERP) estimate for the "market", represented by the average historical risk premium since 1963

$W_d$  = Percent of debt capital in capital structure

$W_e$  = Percent of equity capital in capital structure

The average debt to equity ( $W_d / W_e$ ) ratio of the portfolio is based on the average debt to MVIC for the portfolio since 1963. A debt beta ( $\beta_d$ ) of 0.1 is assumed, which is the average estimated debt beta for the companies included in portfolios 1 through 25 over the years 1963 through 2010 after excluding high-financial-risk companies (high-financial-risk companies are excluded from the base set of companies used in the analysis performed in the *Size Study*).

A debt beta greater than zero indicates debt capital is bearing risk of variability of operating net cash flow in that interest payments and principal repayments may not be made when owed, inferring that tax deductions on the interest expense may not be realized in the period in which the interest is paid.<sup>58</sup> Preferred capital is included with debt capital in measuring the effect of leverage on the risk of equity capital, which is consistent with recent research.<sup>59</sup>

<sup>55</sup> Also updated were Exhibits C-1 through C-8 for the *2010 Duff & Phelps Risk Premium Report*, applying the same (updated) methodology and assumptions. The updated 2010 Exhibits C-1 through C-8 can be downloaded here [www.duffandphelps.com/CostofCapital](http://www.duffandphelps.com/CostofCapital).

<sup>56</sup> Derived from R.S. Harris and J. J. Pringle, "Risk-Adjusted Discount Rates – Extensions from the Average Risk Case," *Journal of Financial Research* (Fall 1985) 237-244. Also see: Arzac, Enrique R., and Lawrence R. Glosten. "A Reconsideration of Tax Shield Valuation." *European Financial Management* (2005): 453-461. For a more complete discussion see chapter eleven in *Cost of Capital: Applications and Examples* 4th ed. by Shannon Pratt and Roger Grabowski, Wiley (2010).

<sup>57</sup> Unlevered betas are often called "asset" betas because they represent the risk of the operations of the business with the risk of financial leverage removed.

<sup>58</sup> For a more complete discussion see Chapter 11 in *Cost of Capital: Applications and Examples* 4th ed. by Shannon Pratt and Roger Grabowski, Wiley (2010).

<sup>59</sup> C.S. Agnes Cheng, C.Z. Liu, K. Newberry, and K.J. Reichelt, "Should Preferred Stock be Classified as a Liability? Evidence from Implied Cost of Equity Capital," working paper (September 2007).

# The Size Study

An example of unlevering the average risk premia from the A exhibits is demonstrated using the information found in Figure 17a, 17b, and 17c (these are abbreviated versions of exhibits A-2, B-2, and C-2, respectively).

The unlevered average risk premium of Portfolio 25 in Exhibit C-2 (Figure 17c) is 10.74 percent, calculated using the following information from Figure 17a, Figure 17b, and Figure 17c:

- The arithmetic average risk premium of Portfolio 25 in Exhibit A-2 (see Figure 17a) is 11.98 percent.
- The debt to market value of equity ( $W_d / W_e$ ) of Portfolio 25 in Exhibit C-2 (see Figure 17c) is 31.9 percent.
- The unlevered sum beta ( $\beta_u$ ) of Portfolio 25 in Exhibit C-2 (see Figure 17c) is 0.99.
- The debt beta ( $\beta_d$ ) is an assumed 0.1, as discussed previously.
- The market premium ( $RP_m$ ) used to perform the analysis in the 2011 Report is the historical ERP from 1963–2010, 4.40%.<sup>60</sup>

To unlever the average (levered) risk premium in Exhibit A-2 (11.98%), substitute these values into the unlevering equation presented earlier:

$$RP_{unlevered} = RP_{levered} - [(W_d / W_e) \times (\beta_u - \beta_d) \times RP_m]$$

$$10.74\% = 11.98\% - ((31.9\% \times (0.99 - 0.1) \times 4.40\%))$$

**Figure 17a: Exhibit A-2 (abbreviated)**

## Companies Ranked by Book Value of Equity

Historical Equity Risk Premium: Average Since 1963  
Data for Year Ending December 31, 2010

### EXHIBIT A-2

Portfolio Rank by Size	Average Book Val. (\$mils.)	Beta (SumBeta) Since '63	Arithmetic Average Risk Premium
1	39,141	0.81	5.08%
2	12,811	0.85	5.30%
← /// →			
25	60	1.27	11.98%

**Figure 17b: Exhibit B-2 (abbreviated)**

## Companies Ranked by Book Value of Equity

Historical Equity Risk Premium: Average Since 1963  
Data for Year Ending December 31, 2010

### EXHIBIT B-2

Portfolio Rank by Size	Average Book Val. (\$mils.)	Premium over CAPM
1	39,141	1.52%
2	12,811	1.57%
← /// →		
25	60	6.41%

**Figure 17c: Exhibit C-2 (abbreviated)**

## Companies Ranked by Book Value of Equity:

### Comparative Risk Characteristics

Data for Year Ending December 31, 2010

### EXHIBIT C-2

Portfolio Rank by Size	Average Book Val. (\$mils.)	Average Debt to Market Value of Equity	Average Unlevered Risk Premium	Average Unlevered Beta
1	39,141	31.39%	4.33%	0.64
2	12,811	39.63%	4.37%	0.64
← /// →				
25	60	31.9%	10.74%	0.99

<sup>60</sup> Derived as the average annual difference between *SBBI* Large Stock total returns (essentially the S&P 500 index) and *SBBI* income returns on long-term Treasury bonds over the time period 1963–2010. Source: Morningstar *EnCorr Analyzer* software.

# The Size Study

## Unlevered Risk Premia—Reconciliation of the A, B and C Exhibits

Reconciliation of the levered and unlevered betas for use in CAPM (found in Exhibits B-2 and C-2, respectively) now reconcile with the levered and unlevered arithmetic average risk premia for the build-up (found in Exhibits A-2 and C-2, respectively), as demonstrated below using the values from the previous example:

*Levered risk premium = Levered beta x Historical market risk premium + Premium over CAPM (i.e. "size premium")*

$$11.98\% = 1.27 \times 4.40\% + 6.41\%$$

*Unlevered risk premium = Levered beta x Historical market risk premium + Premium over CAPM (i.e. "size premium")*

$$10.74\% = 0.99 \times 4.40\% + 6.41\%$$

## Relevering

What if the debt-to-market-value-of-equity ratio ( $W_d/W_e$ ) of the subject company is different than the average ( $W_d/W_e$ ) of the companies making up Portfolio 25 (31.9%)? It may be possible to adjust the (levered) risk premiums over the risk free rate ( $RP_{m+s}$ ) from Exhibits A-1 through A-8 for differences in financial leverage between the subject company and the given guideline portfolio.<sup>61</sup> Again, the average (levered) risk premia presented in Exhibits A-1 through A-8 are unlevered as follows:

$$RP_{unlevered} = RP_{levered} - [(W_d/W_e) \times (\beta_u - \beta_d) \times RP_m]$$

The unlevered risk premia in the C exhibits, which assume a firm is financed 100% with equity and 0% debt, are calculated by unlevering the average risk premia in the A exhibits. In the example, the unlevered risk premium over the risk free rate ( $RP_{m+s, unlevered}$ ) for Portfolio 25 in Exhibit C-2 (10.74%) was calculated by unlevering the average risk premium over the risk free rate ( $RP_{m+s}$ ) for Portfolio 25 in Exhibit A-2 (11.98%). This calculation was performed assuming the 31.9 percent average debt-to-market-value-of-equity ratio ( $W_d/W_e$ ) of the companies making up Portfolio 25.<sup>62</sup> The percentage of debt in the capital structure went from 31.9 percent to 0 percent, and the unlevered risk premia is lower than the levered risk premium.

This formula can be rearranged to "relever":

$$RP_{levered} = RP_{unlevered} + [(W_d/W_e) \times (\beta_u - \beta_d) \times RP_m]$$

If the subject company has a  $W_d/W_e$  ratio that is less (say 20%) than the average  $W_d/W_e$  of the guideline portfolio (31.9%), the unlevered risk premium may be "relevered" at the subject company's lower ratio:

$$11.52\% = 10.74\% + [(20\%) \times (0.99 - 0.1) \times 4.4\%]$$

The subject company has less debt relative to equity than the average company in the guideline portfolio (20% versus 31.9%), and the relevered risk premium is lower than the average levered risk premium of the guideline portfolio (11.52% versus 11.98%). Generally, as the percentage of leverage (debt) in a company's capital structure decreases, risk to equity investors decreases (and vice versa).

<sup>61</sup> If one "relevers" at a debt to equity ( $W_d/W_e$ ) ratio different than the average of  $W_d/W_e$  of the given portfolio, other assumptions may not hold. For example, a debt beta of 0.1 is assumed in the unlevering calculations performed in the Report. If one relevers at a  $W_d/W_e$  ratio that is significantly higher than the average  $W_d/W_e$  ratio of the given guideline portfolio, a higher debt beta than 0.1 may be expected, all things held the same.

<sup>62</sup> As found in Exhibit C-2. It is important to understand that each of the A, B, and C exhibits is sorted by different size criteria. For instance, the base set of companies used to perform the analyses in the Size Study is sorted by "book value of equity", and then used to calculate the different data and information presented in the A-2, B-2, and C-2 exhibits. Citing the present unlevering/relevering example, the average debt-to-market-value-of-equity ratio ( $W_d/W_e$ ) of the smallest companies (Portfolio 25) as sorted by "book value of equity" is found in Exhibit C-2, while the average debt-to-market-value-of-equity ratio ( $W_d/W_e$ ) of the smallest companies (Portfolio 25) as sorted by "total assets" is found in Exhibit C-5.

# The Risk Study

The *Risk Study* is an extension of the *Size Study*. The main difference between the *Risk Study* and the *Size Study* is that while the *Size Study* analyzes the relationship between size and return, the *Risk Study* analyzes the relationship between fundamental risk measures (based on accounting data) and return. These are called “fundamental” measures of company risk to distinguish these risk measures from a stock market-based measure of equity risk such as beta. A variety of academic studies have examined the relationship between financial statement data and various aspects of business risk.<sup>95</sup> Research has shown that measures of earnings volatility can be useful in explaining credit ratings, predicting bankruptcy, and explaining the CAPM beta.

As in the *Size Study*, 25 portfolios are created, but instead of being ranked by eight alternative measures of size as is done in the *Size Study*, the *Risk Study* portfolios are ranked by three fundamental risk measures: five-year average operating income margin, the coefficient of variation in operating income margin, and the coefficient of variation in return on book equity.<sup>96, 97</sup> The first statistic measures profitability and the other two statistics measure volatility of earnings. All three measures use average financial data for the five years preceding the formation of annual portfolios.

## Size and Risk

Traditionally, valuation professionals have used company size as a factor in determining discount rates for smaller companies. Small companies are believed to have higher required rates of return than large companies because small companies are inherently riskier. The historical data (as published in the *Duff & Phelps Risk Premium Report*, as well as in the *SBBi*), verify that small companies have, in fact, earned higher rates of return over long-run periods.

It has been pointed out in the financial literature that researchers may be mixing a “size” effect with a “risk” effect when measuring company size by market value,<sup>98</sup> but market value is not just a function of “size”; it is also a function of the discount rate. In other words, some companies might be small because they are risky, rather than risky because they are small. The *Risk Study* goes beyond size and investigates the relationship between equity returns and fundamental risk measures. Does the evidence support the claim that smaller companies inherently have greater risk? The *Risk Study* analyzes this question, and demonstrates that as company size decreases, measures of risk calculated from financial statement data do, as a matter of fact, tend to increase.<sup>99</sup> The data clearly shows that as fundamental risk *increases* in the form of lower profitability or greater variability of earnings, the return over the risk free rate tends to *increase*. These relationships are summarized in Figure 38.

**Figure 38: Operating Margin (i.e. “profitability”) and Variability of Earnings versus Risk.**



<sup>95</sup> A survey of the academic research can be found in *The Analysis and Use of Financial Statements*, 3rd edition, White et al., Wiley (2003), chapter 18.

<sup>96</sup> Coefficient of variation is defined here as the standard deviation divided by the mean.

<sup>97</sup> For a detailed discussion of portfolio creation methodology, see “Portfolio Methodology” on page 9.

<sup>98</sup> “A Critique of Size Related Anomalies,” Jonathan Berk, *Review of Financial Studies*, vol. 8, no. 2 (1995).

<sup>99</sup> A similar point was made by Barry Goodman in a presentation at the October 1997 American Society of Appraisers’ *Advanced Business Valuation Conference* in San Francisco.

# The Risk Study

Previously, it was demonstrated in the *Size Study* that there is a clear *inverse* relationship between size and historical rates of return (as size decreases, returns tend to increase; see Graph 2 on page 18). In the *Risk Study*, the data show a clear *direct* relationship between accounting-data-based fundamental risk measures and historical rates of return (as fundamental risk increases, returns tend to increase).

In Graph 3, as fundamental risk *increases* (from left to right), the average annual return over the study time horizon (1963–2010) tends to *increase* for each of the three fundamental risk measures.

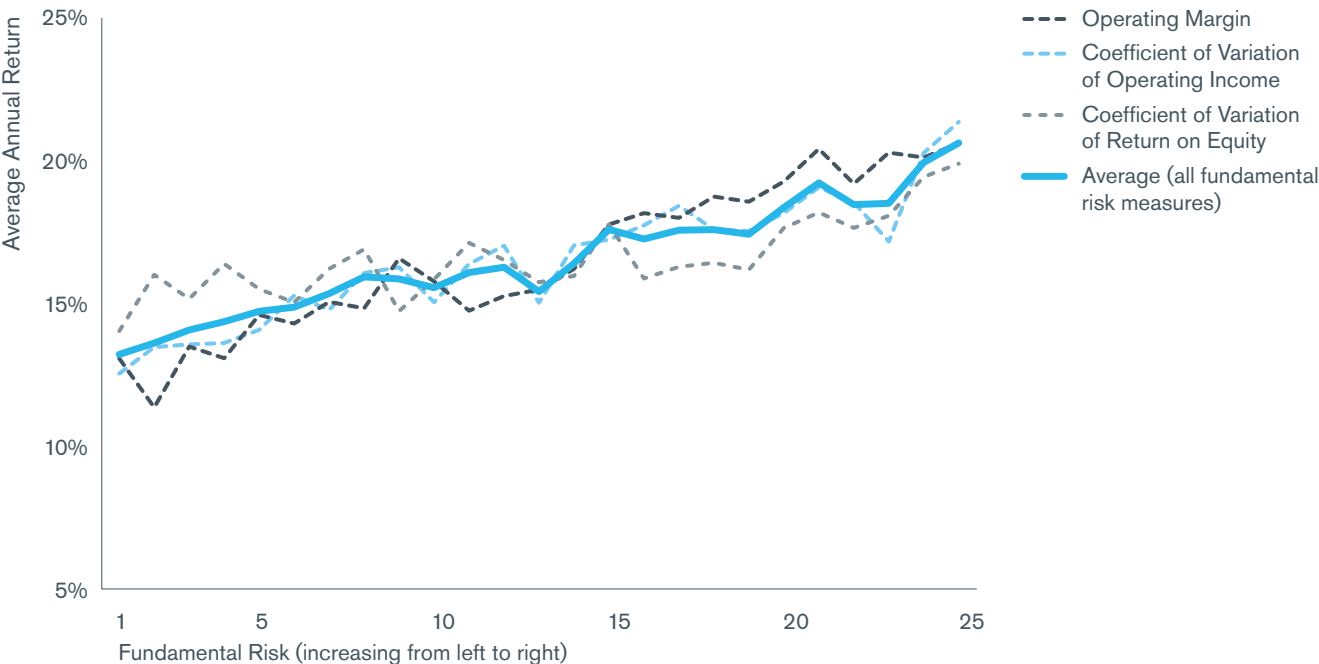
For example, in the *2011 Report*, the average annual return of the portfolios made up of companies with the lowest risk as measured by each of the three fundamental risk measures was 13.2 percent, while the average annual return of the portfolios made up of companies with the highest risk as measured by each of the three fundamental risk measures was 20.6 percent.

## Reasons for Using Fundamental Measures of Risk in Addition to Measures of Size

First, certain measures of size (such as market value of equity) may be imperfect measures of the risk of a company's operations in some situations. For example, a company with a large and stable operating margin may have a small and unstable market value of equity if it is highly leveraged. In this case the risk of the underlying operations is *low* while the risk to equity is *high*.

Second, while small size may indicate greater risk, some small companies may maintain near economic monopolies by holding a geographic niche or market niche such that their true riskiness is *less* than what would be indicated by their size.

Graph 3: Average Annual Return, Three Measures of Fundamental Risk  
1963–2010





# The Risk Study

Alternatively, while larger size (as measured by sales, for example) may indicate less risk, some companies may be riskier than the average of companies with similar sales. For example, assume the subject company was expecting to emerge from reorganization following bankruptcy. The risk premium appropriate for this company may be more accurately imputed from the pro-forma operating profit (after removing non-recurring expenses incurred during the bankruptcy) than from its size as measured by sales. In other words, the subject company may be riskier than companies with similar sales volumes.

Use of fundamental accounting measures of risk allows for direct assessment of the riskiness of the subject company. For example, if the appropriate equity risk premium for the subject company when measuring risk by one or more fundamental risk measures is different than the equity risk premium based on size measures, this difference may be an indication of the "company-specific risk" of the subject company.<sup>100</sup>

## Presentation of the Results

The *Risk Study's* D exhibits present 25 portfolios ranked by three fundamental risk factors (based on accounting data). These fundamental risk factors are described in Table 4.<sup>101</sup>

**Table 4: Three Measures of Fundamental Risk in the *Risk Study's* D Exhibits**

### Exhibits D-1

Operating Margin: The mean operating income for the prior five years divided by the mean sales for the prior five years. Operating income is defined as sales minus cost of goods sold plus selling, general, and administrative expenses plus depreciation. Note that this composite ratio is usually very close to a simple average of the annual ratios of operating income to sales, except in extreme cases generally involving companies with high growth rates.

### Exhibit D-2

Coefficient of Variation of Operating Margin: The standard deviation of operating margin over the prior five years divided by the average operating margin for the same years. Note that for calculating this coefficient, average operating margin is a simple average of the annual ratios of operating income to sales rather than the composite ratio used in exhibit D-1.

### Exhibit D-3

Coefficient of Variation of Return on Book Value of Equity: The standard deviation of return on book equity for the prior five years divided by the mean return on book equity for the same years. Return on book equity is defined as net income before extraordinary items minus preferred dividends divided by book value of common equity.

Each of the *Risk Study's* exhibits D-1 through D-3 displays one line of data for each of the 25 fundamental-risk-ranked portfolios. The D exhibits include the twelve statistics outlined in Table 5. For comparative purposes, the average returns from the *S&B*I series for large companies (essentially the S&P 500 Index), small companies, and long-term government bond income returns for the period 1963 through the latest year are also reported on each exhibit.<sup>102</sup>

**Table 5: Statistics Reported for 25 fundamental-risk-ranked portfolios in the *Risk Study's* D Exhibits**

<ul style="list-style-type: none"> <li>The average of the sorting criteria for the latest year (e.g., the average operating margin for the latest five years before 2010). In the <i>2011 Report</i>, the "latest year" is 2010. Note that the reported average risk statistics in exhibits D-1, D-2, and D-3 are not the same numbers as reported in exhibits C-1 through C-8. In exhibits C-1 through C-8, the reported statistics are calculated for portfolios of companies grouped according to size and are averages since 1963. In exhibits D-1, D-2, and D-3, the reported statistics are calculated for portfolios grouped according to risk, independent of the "size" of the companies, and are not averages since 1963</li> </ul>	<ul style="list-style-type: none"> <li>Geometric average historical equity return since 1963.</li> </ul>
<ul style="list-style-type: none"> <li>Log (base-10) of the average of the sorting criteria.</li> </ul>	<ul style="list-style-type: none"> <li>Arithmetic average historical equity return since 1963.</li> </ul>
<ul style="list-style-type: none"> <li>The number of companies in each portfolio in the latest year. In the <i>2011 Report</i>, the "latest year" is 2010.</li> </ul>	<ul style="list-style-type: none"> <li>Arithmetic average historical risk premium over long-term Treasuries (average return on equity in excess of long-term Treasury bonds) since 1963. (<math>RP_{m+u}</math>)</li> </ul>
<ul style="list-style-type: none"> <li>Beta calculated using the "sum beta" method applied to monthly returns for 1963 through the latest year (see the <i>2011 S&amp;B</i>I Valuation Yearbook pp. 77-78 for a description of the "sum beta" method).</li> </ul>	<ul style="list-style-type: none"> <li>Unlevered arithmetic average historical risk premium over long-term Treasuries (average return on equity in excess of long-term Treasury bonds) since 1963. (<math>RP_{m+u, unlevered}</math>)</li> </ul>
<ul style="list-style-type: none"> <li>Unlevered beta calculated using the "sum beta" method applied to monthly returns for 1963 through the latest year.</li> </ul>	<ul style="list-style-type: none"> <li>"Smoothed" average historical risk premium over long-term Treasuries (average return on equity in excess of long-term Treasury bonds) since 1963: the fitted premium from a regression with the historical "risk premium over long-term Treasuries" as dependent variable and the logarithm of the average sorting criteria as independent variable. (<math>RP_{m+u}</math>)</li> </ul>
<ul style="list-style-type: none"> <li>Standard deviation of annual historical equity returns.</li> </ul>	<ul style="list-style-type: none"> <li>Average Debt as a percent of the MVC since 1963.</li> </ul>

<sup>100</sup> *Valuing a Business*, 4th ed., Pratt et al, McGraw-Hill (2000), p 181. Company-specific risk factors can include concentration of customer base, key person dependence, key supplier dependence, or any number of other factors that are unique to the subject company.

<sup>101</sup> For a detailed description of the Standard and Poor's *Compustat* data items used in the *Duff & Phelps Risk Premium Report*, please see Appendix A (in the full *Report*).

<sup>102</sup> Source: Morningstar *EnCorr Analyzer* software.

# The Risk Study

## Overview of Methods Used to Estimate Cost of Equity Capital using the Risk Study

The *Risk Study* provides one method of estimating COE for a subject company, Buildup 3, plus one method for estimating unlevered COE (the cost of equity capital assuming a firm is financed 100% with equity and 0% debt).

These methods are summarized below in equation format, and summarized in Figure 39 in graphical “building blocks” format.

### 1) Buildup 3

$$COE_{Buildup\ 3} = (Risk\ Free\ Rate) + (Risk\ Premium\ Over\ Risk\ Free\ Rate) + (Equity\ Risk\ Premium\ Adjustment)$$

Example 5a: using Guideline portfolios: page 46

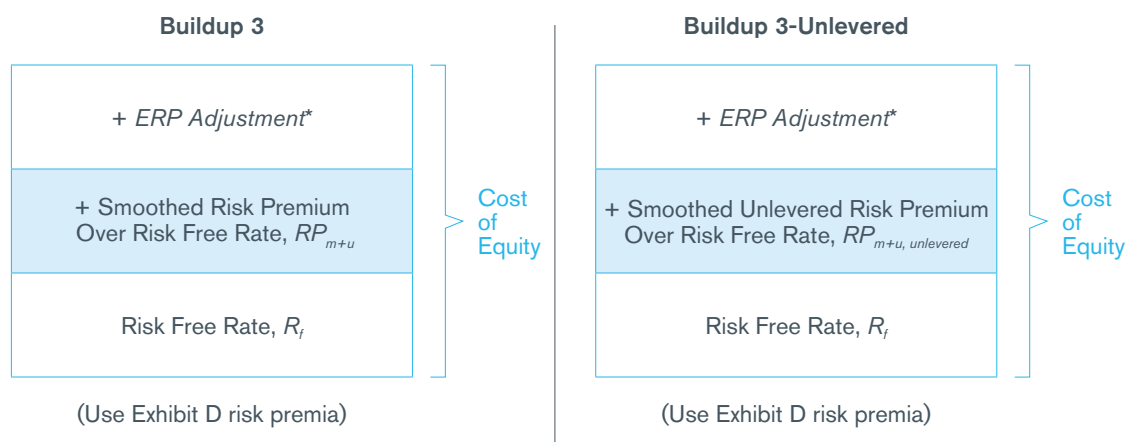
Example 5b: using regression equations: page 49

### 2) Buildup 3-Unlevered

$$COE_{Buildup\ 3-Unlevered} = (Risk\ Free\ Rate) + (Unlevered\ Risk\ Premium\ Over\ Risk\ Free\ Rate) + (Equity\ Risk\ Premium\ Adjustment)$$

Example 6: using Guideline portfolios: page 74 (in the full *Report*)

Figure 39: Two Methods of Estimating Cost of Equity Capital with the *Risk Study*<sup>103</sup>



\*ERP Adjustment: The difference between the historical (1963–2010) equity risk premium (ERP) and a user of the *Duff & Phelps Report*'s own forward ERP estimate:

$$ERP\ Adjustment = User's\ ERP - Historical\ (1963-2010)\ ERP$$

The ERP Adjustment is made only in the “Buildup 1”, “Buildup 1-Unlevered”, “Buildup 1-High-Financial-Risk”, “Buildup 3”, and “Buildup 3-Unlevered” methods. Please refer to the individual examples provided for these models for more information.

NOTE: This section includes an example of using the *Report*'s risk premia data to estimate cost of equity capital using the “Buildup 3” method, plus an overview of the “C” data exhibits, which provide a “link” between the *Size Study* and the *Risk Study*. The C exhibits can be used to gauge company-specific risk adjustments.

A complete example for using the *Report*'s risk premia to estimate cost of equity capital using the “Buildup 3-Unlevered” method is available in the full version of the 2011 *Report*.

<sup>103</sup> The relative sizes of the “building blocks” in Figure 39 do not necessarily represent the relative size of the various inputs. Also note that the names given to the models in the *Risk Premium Report* (e.g. “Buildup 1”, “Buildup 2”, “Buildup 3”, etc) are naming conventions used within the *Report* to make referring to the different methods easier.

# The Risk Study

The three risk measures outlined in Table 4 (page 41) can be used with either of the two methods of estimating COE provided by the *Risk Study*. It is important to note that the subject company information necessary to calculate all of these measures may not be available. In these cases, it is generally acceptable to use the fundamental risk measures that *are* available. It is recommended, however, that *Report* users calculate available risk measures for the subject company using at least the three most recent years of data, and the five most recent years of data for best results.

## Gathering Accounting Information to Calculate Fundamental Risk Measures

The first step in using the *Risk Study* to estimate cost of equity capital (COE) is to gather the accounting-based information for the subject company needed to calculate the three fundamental risk measures analyzed in the *Risk Study*.

- To calculate “operating margin” and “coefficient of variation of operating margin”, net sales and operating income are needed.
- To calculate “coefficient of variation of ROE”, book value and net income before extraordinary items are needed.

The accounting information for the last 5 years needed to calculate the three fundamental risk measures for a hypothetical subject company is summarized in Figure 40a and Figure 40b.

**Figure 40a: Subject Company Operating Margin and Coefficient of Variation of Operating Margin (used in all examples)**

	2010	2009	2008	2007	2006
Net Sales	\$900	\$800	\$850	\$750	\$900
Operating Income	\$150	\$120	\$130	\$80	\$140
Operating Margin	16.7% = \$150/\$900	15.0% = \$120/\$800	15.3% = \$130/\$850	10.7% = \$80/\$750	15.6% = \$140/\$900
Standard Deviation of Operating Margin	2.3%				
Average Operating Margin	14.6%				
Coefficient of Variation of Operating Margin	15.8% = 2.3%/14.6%				

**Figure 40b: Subject Company Coefficient of Variation of ROE (used in all examples)**

	2010	2009	2008	2007	2006
Book Value	\$820	\$710	\$630	\$540	\$500
Net Income before extraordinary items	\$110	\$80	\$90	\$40	\$100
Return on Book Equity (ROE)	13.4% = \$110/\$820	11.3% = \$80/\$710	14.3% = \$90/\$630	7.4% = \$40/\$540	20.0% = \$100/\$500
Standard Deviation of ROE	4.6%				
Average ROE	13.3%				
Coefficient of Variation of ROE	34.7% = 4.6%/13.3%				

# The Risk Study

The hypothetical subject company has an average operating margin of 14.6 percent, a coefficient of variation of operating margin of 15.8 percent, and a coefficient of variation of ROE of 34.7 percent, as summarized in Figure 41.<sup>104</sup>

**Figure 41: Subject Company Fundamental Risk Characteristics (used in all Examples)**

	Risk Measure	Appropriate Exhibit	
		Buildup 3	Buildup 3-Unlevered
Average Operating Margin	14.6%	D-1	D-1
Coefficient of Variation of Operating Margin	15.8%	D-2	D-2
Coefficient of Variation of ROE	34.7%	D-3	D-3

Figure 41 also includes the data exhibits in which the appropriate risk premia for each of the size measures can be found. For example, for use in the Buildup 3 method, risk premia over the risk free rate ( $RP_{m+u}$ ) for “coefficient of variation of operating margin” are found in Exhibit D-2. For use in the Buildup 3-Unlevered method, unlevered risk premia over the risk free rate ( $RP_{m+u, unlevered}$ ) for “coefficient of variation of operating margin” are also found in Exhibit D-2.

In each of the following examples of using the *Risk Study* to estimate COE, the subject company risk measures summarized in Figure 41 will be used (average operating margin of 14.6 percent, for instance, will be used in all examples).

## Estimating Cost of Equity Capital Using the “Buildup 3” Method

### Buildup 3



(Use Exhibit D risk premia)

The buildup method is an additive model commonly used for calculating the required rate of return on equity. As the name implies, successive “building blocks” are summed, each representing the additional risk inherent to investing in alternative assets. An example of this is the extra return (i.e. “premium”), that investors demand for investing in stocks versus investing in a riskless security.<sup>105</sup>

### Risk Premium Over Risk Free Rate, $RP_{m+u}$

The risk premia developed in the *Risk Study* ( $RP_{m+u}$ ) take the form of “risk premia over the risk free rate”, but are slightly different from the risk premia over the risk free rate ( $RP_{m+s}$ ) that are developed in the *Size Study*, which are a measure of risk in terms of the total effect of market risk and size risk.<sup>106</sup> Because operating efficiencies (or lack thereof) of the subject company are being captured by the use of accounting-based risk measures, the difference in the average rate of return for each risk-based portfolio over the sample period and the income return earned of long-term Treasury bonds (using *S&P* data) is a measure of risk in terms of the total effect of market risk, and company-specific risk ( $RP_{m+u}$ ).<sup>107</sup> The result is a clear direct relationship between fundamental risk and premium over long-term bond yields. As fundamental risk increases, the return over the risk free rate (i.e. “excess return”) tends to increase.

The  $RP_{m+u}$  risk premia can be added to the risk free rate ( $R_f$ ) to estimate cost of equity capital (COE) using the Buildup 3 method.

<sup>104</sup> Coefficient of variation is defined here as the standard deviation divided by the mean. For example (using a Microsoft Excel formula), the coefficient of variation of operating margin of the hypothetical subject company used in all examples =  $STDEV(16.7, 15.0, 15.3, 10.7, 15.6)/AVERAGE(16.7, 15.0, 15.3, 10.7, 15.6)$ .

<sup>105</sup> Throughout this report the risk free asset is represented by the yield on a 20-year constant maturity Treasury bond.

<sup>106</sup> For a detailed discussion of how premia over the risk free rate are calculated, see “The Difference Between ‘Risk Premium Over the Risk Free Rate’ and ‘Risk Premium Over CAPM’” on page 22.

<sup>107</sup> Because these premia have an embedded measure of market (i.e. “beta”) risk, these premia are appropriate for use in “buildup” methods that do not otherwise include a measure of market risk, but are not appropriate for use in models (e.g. CAPM) that already have a measure of market (beta) risk. *Risk Study* risk premia over the risk free rate ( $RP_{m+u}$ ) are published in Exhibits D-1, D-2, and D-3 of the *Risk Premium Report*.

# The Risk Study

## The “Buildup 3” Equation

As an alternative to the basic buildup equation (see page 29 in the full *Report*), one can use the *Risk Study* to develop a risk premium for the subject company for which  $RP_m$  (the market premium) and  $RP_u$  (the company-specific risk premium) are combined into a single premium,  $RP_{m+u}$ . The basic buildup equation therefore becomes:

$$E(R_i) = R_f + RP_{m+u}$$

where:

$E(R_i)$  = Expected rate of return on security i (this is “cost of equity capital”, or “COE”)

$R_f$  = risk free rate as of the valuation date (typically a long-term US Treasury bond yield)

$RP_{m+u}$  = risk premium for the subject company for which  $RP_m$  (the market premium) and  $RP_u$  (the company-specific risk premium) are combined into a single premium.

One final important modification of the basic buildup formula is needed: the “equity risk premium (ERP) adjustment”. The equity risk premium adjustment is made to reconcile the historical data presented in the *Risk Premium Report* with the forward-looking ERP chosen by the individual analyst as of valuation date.<sup>108</sup>

The *ERP Adjustment* is simply the difference between the user’s own forward-looking ERP and the historical 1963–2010 ERP (4.40%).<sup>109</sup> For example, many users of the *Report* use the *Duff & Phelps Recommended ERP*, which was 5.5 percent at the end of 2010).<sup>110, 111</sup> In this case, the *ERP Adjustment* would be 1.1 percent (5.5%–4.4%).

Adding the *ERP Adjustment* to the basic buildup formula produces the full equation for the “Buildup 3” method:

$$COE_{Buildup\ 3} = R_f + RP_{m+u} + ERP\ Adjustment$$

The Buildup 3 method is a straightforward way of estimating cost of equity capital (COE) using the historical “risk premiums over the long-term risk-free rate” ( $RP_{m+u}$ ) presented in exhibits D-1 through D-3. It is important to understand that because the risk premia presented in the D exhibits have an embedded measure of market (i.e. “beta”) risk, they are appropriate only for use in “buildup” methods that do not otherwise include a measure of market risk; these premia are *not* appropriate for use in models (e.g. CAPM) that already have a measure of market (beta) risk.<sup>112</sup>

As noted previously, the *Risk Premium Report* provides two ways for analysts to match their subject company’s size (or risk) characteristics with the appropriate smoothed premia from the data exhibits: the “guideline portfolio” method and the “regression equation” method.<sup>113</sup> In general, the regression equation method is preferred because this method allows for interpolation between the individual guideline portfolios, although the guideline portfolio method is less complicated, and more direct. Examples of both the guideline portfolio method and the regression equation method follow, starting with the simpler guideline portfolio method.

<sup>108</sup> The “ERP Adjustment” is necessary in the *Size Study*’s “Buildup 1” method and “Buildup 1-Unlevered” method, and in the *Risk Study*’s “Buildup 3” method and “Buildup 3-Unlevered” method. See page 30 in the full *Report* for more a detailed discussion of the equity risk premium adjustment.

<sup>109</sup> Calculated as the annual *S&P* Large Company Stock (essentially the S&P 500 Index) return minus the average annual long-term *S&P* government bond income return over the time horizon 1963–2010. Source: Morningstar *EnCorr Analyzer* software.

<sup>110</sup> For more information on the equity risk premium, see *Cost of Capital: Applications and Examples* 4th ed., by Shannon P. Pratt and Roger J. Grabowski (John Wiley & Sons, Inc., 2010), Chapter 9, “Equity Risk Premium”, pages 115–158.

<sup>111</sup> See Roger J. Grabowski, “Developing the Cost of Equity Capital: Risk-Free Rate and ERP During Periods of ‘Flight to Quality’”. This paper will appear in the *Business Valuation Review* and can also be downloaded at Duff & Phelps’ Cost of Capital site at [www.duffandphelps.com/CostofCapital](http://www.duffandphelps.com/CostofCapital).

<sup>112</sup> Please refer to page 28 for examples illustrating how to use size premia in conjunction with CAPM to estimate COE.

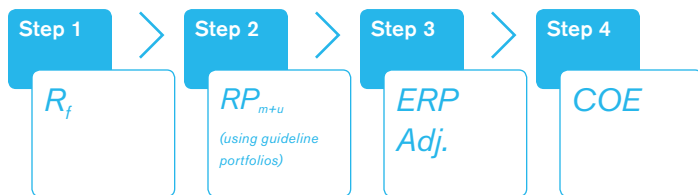
<sup>113</sup> See pages 14–16 for a detailed explanation of the differences between the guideline portfolio method and the regression equation method.

# The Risk Study

## Example 5a: Buildup 3 Method (using guideline portfolios)

Three pieces of information are needed to estimate the cost of equity capital using the Buildup 3 method using “guideline portfolios”: a risk free rate ( $R_f$ ), a risk premium over the risk free rate ( $RP_{m+u}$ ), and an *ERP Adjustment* (if necessary). All of the information needed is summarized in Figure 42.

**Figure 42: Information Needed to Estimate COE Using Buildup 3 and Guideline Portfolios**



This example utilizes the long-term risk free rate ( $R_f$ ) and the *ERP Adjustment* established in a previous example (the *Size Study's* Buildup 1 method using “guideline portfolios”; see page 31 in the full *Report*). This mirrors the fact that for any given valuation engagement the same risk free rate and ERP will generally be used in each of the models presented by the individual analyst. Please note that for any given valuation engagement these inputs may (and probably will) be different than the ones used in the examples. The only missing ingredients needed to estimate COE are the risk premia over the risk free rate ( $RP_{m+u}$ ), as summarized in Figure 43.

**Step 1, Risk Free Rate ( $R_f$ ):** The risk free rate is typically a long-term US Treasury bond yield as of the valuation date. This example utilizes the assumed long-term treasury yield of 4.1 percent established in Example 1a (on page 31 in the full *Report*).

**Step 2, Risk Premium Over Risk Free Rate ( $RP_{m+u}$ ):** Match the various fundamental risk measures of the subject company with the guideline portfolios composed of companies of similar fundamental risk measures in Exhibits D-1 through D-3, and identify the corresponding smoothed average risk premium.

The subject company in this example has an average operating margin of 14.6 percent, and the appropriate data exhibit is Exhibit D-1 (see Figure 41 on page 44). An abbreviated version of Exhibit D-1 is shown in Figure 44. Of the 25 portfolios, the portfolio that has an average operating margin *closest* to the subject company's 14.6 percent is Portfolio 9 (14.37%). The corresponding smoothed average risk premium ( $RP_{m+u}$ ) is 8.72 percent (8.7%, rounded).

**Figure 43: Needed—Smoothed Risk Premia Over the Risk Free Rate ( $RP_{m+u}$ ) Using Guideline Portfolios**

				Step 1		Step 2		Step 3		Step 4	
				Risk Free Rate, $R_f$		Smoothed Premium Over Risk Free Rate, ( $RP_{m+u}$ )		<i>ERP Adjustment</i>		COE	
	Risk Measure	Appropriate Exhibit	Guideline Portfolio								
Operating Margin	14.6%	D-1	?	4.1%	+	?	+	1.1%	=		
Coefficient of Variation of Operating Margin	15.8%	D-2	?	4.1%	+	?	+	1.1%	=		
Coefficient of Variation of ROE	34.7%	D-3	?	4.1%	+	?	+	1.1%	=		
Mean (average) values				4.1%	+		+	1.1%	=		
Median (typical) values				4.1%	+		+	1.1%	=		

# The Risk Study

Match all of the subject company's risk measures in this fashion. For example, the subject company in this example has a "coefficient of variation of operating margin" of 15.8 percent. Of the 25 guideline portfolios in Exhibit D-2 (not shown here), the portfolio that has a coefficient of variation of operating margin *closest* to the subject company's 15.8 percent coefficient of variation of operating margin is Portfolio 13 (16.2%). The corresponding smoothed average risk premium is 9.5 percent. In the case of the third risk measure, the subject company has a "coefficient of variation of ROE" of 34.7 percent. Of the 25 guideline portfolios in Exhibit D-3 (not shown here), the portfolio that has a coefficient of variation of ROE *closest* to the subject company's 34.7 percent coefficient of variation of ROE is Portfolio 14 (33.7%). The corresponding smoothed average risk premium is 9.5 percent.

At this point, all of the available risk measures for the subject company have been matched to the closest guideline portfolio in the appropriate exhibit, and the corresponding smoothed average risk premium has been identified for each, and Step 2 is complete.

**Step 3, Equity Risk Premium (ERP) Adjustment:** The *ERP Adjustment* is needed to account for any difference in the user's own ERP estimate and the historical (1963–2010) ERP. This example utilizes the *ERP Adjustment* (1.1%) established in Example 1a (page 31 in the full *Report*).

**Figure 44: Exhibit D-1 (abbreviated)**

## Companies Ranked by Operating Margin

Historical Equity Risk Premium: Average Since 1963

Data for Year Ending December 31, 2010

Portfolio Rank	Average Operating Margin	Log of Average Op Margin	Number as of 2010	Beta (SumBeta) Since '63	Standard Deviation of Returns	Arithmetic Average Return	Arithmetic Average Risk Premium	Arithmetic Average Unlevered Risk Premium	Smoothed Average Risk Premium	Average Debt/MVIC
1	38.99%	-0.41	57	0.87	17.00%	13.08%	6.17%	5.32%	4.98%	25.51%
2	29.61%	-0.53	58	0.82	17.51%	11.37%	4.40%	3.67%	3.31%	28.26%
3	25.17%	-0.60	51	0.85	17.43%	13.50%	6.38%	5.71%	5.02%	26.83%
4	22.02%	-0.66	58	0.94	16.98%	13.09%	6.10%	5.33%	7.12%	23.23%
5	19.69%	-0.71	52	0.98	18.47%	14.61%	7.70%	6.80%	7.04%	19.85%
6	17.89%	-0.75	53	1.06	17.88%	14.31%	7.40%	6.67%	7.00%	17.37%
7	16.85%	-0.77	47	1.10	19.22%	15.06%	8.16%	7.38%	8.12%	17.81%
8	15.79%	-0.80	52	1.11	20.07%	14.84%	7.94%	7.12%	8.38%	18.39%
9	14.37%	-0.84	57	1.16	20.22%	16.59%	9.69%	8.78%	8.72%	19.37%
10	13.43%	-0.87	52	1.16	20.80%	15.78%	8.88%	7.92%	8.67%	20.21%
///										
24	3.78%	-1.42	81	1.29	26.33%	20.13%	13.22%	11.80%	13.71%	30.84%
25	2.18%	-1.66	113	1.29	28.90%	20.62%	13.72%	12.12%	15.18%	30.57%

Note: Some values intentionally blurred.

# The Risk Study

**Step 4, Estimate Cost of Equity (COE):** With the completion of Steps 1 through 3, the information needed to estimate a base cost of equity capital using the Buildup 3 method (using guideline portfolios) is now completed. The risk premiums over the risk free rate ( $RP_{m+u}$ ) can be added to the risk-free rate ( $R_f$ ) and the *ERP Adjustment* to estimate an indicated cost of equity capital (COE) for the subject company, as illustrated in Figure 45.

The range of COE estimates for the hypothetical subject company in this example is 13.9 percent to 14.7 percent, with an average of 14.4 percent, and a median of 14.7 percent. The mean represents the average estimate, but the mean can be unduly influenced by very large or very small “outliers”. For this reason, the median estimate is generally preferred to the mean. The median estimate tends to not be as heavily influenced by very large or very small outliers, and can be considered a measure of the “typical” estimate in the group.

Use of fundamental accounting measures of risk allows for direct assessment of the riskiness of the subject company. For example, if the appropriate equity risk premium for the subject company when measuring risk by one or more fundamental risk measures is different than the equity risk premium based on size measures, this difference may be an indication of the “company-specific risk” of the subject company.<sup>114</sup>

**Figure 45: Buildup 3 COE Inputs (using guideline portfolios)**

				Step 1		Step 2		Step 3		Step 4
				Risk Free Rate, $R_f$		Smoothed Premium Over Risk Free Rate, ( $RP_{m+u}$ )		ERP Adjustment		COE
Operating Margin	14.6%	D-1	9	4.1%	+	8.7%	+	1.1%	=	13.9%
Coefficient of Variation of Operating Margin	15.8%	D-2	13	4.1%	+	9.5%	+	1.1%	=	14.7%
Coefficient of Variation of ROE	34.7%	D-3	14	4.1%	+	9.5%	+	1.1%	=	14.7%
Mean (average) values				4.1%	+	9.2%	+	1.1%	=	14.4%
Median (typical) values				4.1%	+	9.5%	+	1.1%	=	14.7%

<sup>114</sup> *Valuing a Business*, 4th ed., Pratt et al, McGraw-Hill (2000), p 181. Company-specific risk factors can include concentration of customer base, key person dependence, key supplier dependence, or any number of other factors that are unique to the subject company.



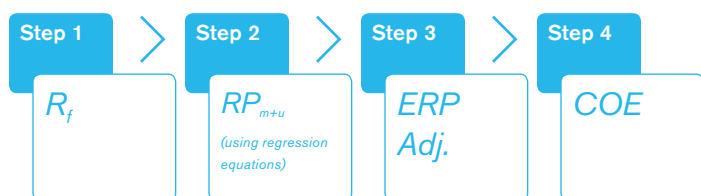
# The Risk Study

## Example 5b: Buildup 3 Method (using regression equations)

When the subject company risk measures do not exactly match the respective average risk measure of the guideline portfolios, the data exhibits provide a straightforward way to interpolate an “exact” risk premium over the risk free rate between guideline portfolios using the “regression equation” method.

The *only* difference between estimating cost of equity capital (COE) using the Buildup 3 method using “guideline portfolios” (as in the previous example) and COE using the Buildup 3 method using “regression equations” is how the risk premium over the risk free rate ( $RP_{m+u}$ ) are identified in Step 2.

**Figure 46: Steps Needed to Estimate COE Using Buildup 3 and Regression Equations**



In the previous example, the smoothed average risk premia *published* in the report for the appropriate guideline portfolios were used to estimate COE.<sup>115</sup> In this example, however, the regression equations found in each of the data exhibits will be used to calculate “custom” interpolated risk premia, based upon the specific risk measures of the subject company.

Please note that this example utilizes the long-term risk free rate ( $R_f$ ) and the *ERP Adjustment* established in a previous example (the *Size Study*’s Buildup 1 method using “guideline portfolios”; see page 31 in the full *Report*). This mirrors the fact that for any given valuation engagement the same risk free rate and ERP will generally be used in each of the models presented by the individual analyst. Please note that for any given valuation engagement these inputs may (and probably will) be different than the ones used in the examples. The only missing ingredients needed to estimate COE are the risk premia over the risk free rate ( $RP_{m+u}$ ), as summarized in Figure 47.

**Figure 47: Buildup 3 COE Inputs (using regression equations)**

			Step 1		Step 2		Step 3		Step 4	
			Risk Free Rate, $R_f$		Smoothed Premium Over Risk Free Rate, ( $RP_{m+u}$ )		ERP Adjustment		COE	
	Risk Measure	Appropriate Exhibit								
Operating Margin	14.6%	D-1	4.1%	+	?	+	1.1%	=		
Coefficient of Variation of Operating Margin	15.8%	D-2	4.1%	+	?	+	1.1%	=		
Coefficient of Variation of ROE	34.7%	D-3	4.1%	+	?	+	1.1%	=		
Mean (average) values			4.1%	+		+	1.1%	=		
Median (typical) values			4.1%	+		+	1.1%	=		

<sup>115</sup> The smoothed risk premia published in the *Risk Premium Report* are based upon the average size (or risk) measure of each of the respective guideline portfolios.

# The Risk Study

**Step 1, Risk Free Rate ( $R_f$ ):** The risk free rate is typically a long-term US Treasury bond yield as of the valuation date. This example utilizes the assumed long-term treasury yield of 4.1 percent established in Example 1a (page 31 in the full *Report*).

**Step 2, Risk Premium Over the Risk Free Rate ( $RP_{m+u}$ ):** The hypothetical subject company in this example has an average operating margin of 14.6 percent, and the appropriate data exhibit is Exhibit D-1 (see Figure 41)<sup>116</sup>. In this case one would expect that the smoothed average risk premium over the risk free rate ( $RP_{m+u}$ ) would fall somewhere between 8.36 percent (the smoothed risk premium over the risk free rate for Portfolio 8) and 8.72 percent (the smoothed risk premium over the risk free rate for Portfolio 9), as illustrated in Figure 48:

An easy way to calculate a custom interpolated risk premium over the risk free rate ( $RP_{m+u}$ ) "in between" Portfolio 8 and Portfolio 9 is by using the regression equations provided for this purpose in each of the data exhibits. The regression equations are located in the same spot in each of the data exhibits (see Figure 4 on page 16).<sup>117</sup>

The regression equation provided in Exhibit D-1, which includes 25 portfolios ranked by operating margin<sup>118</sup>, is:

$$\text{Smoothed Premium} = 1.453\% - 8.622\% * \text{Log(Operating Margin)}$$

To calculate an interpolated risk premium for the subject company, substitute the subject company's 14.6 percent operating margin into the regression equation as follows<sup>119</sup>:

$$\text{Smoothed Premium} = 1.453\% - 8.622\% * \text{Log}(14.6\%)$$

$$8.6\% = 1.453\% - 8.622\% * (-0.84)$$

**Figure 48: Exhibit D-1 (abbreviated)**

## Companies Ranked by Operating Margin

Historical Equity Risk Premium: Average Since 1963

Data for Year Ending December 31, 2010

Portfolio Rank	Average Operating Margin	Log of Average Op Margin	Number as of 2010	Beta (SumBeta) Since '63	Standard Deviation of Returns	Arithmetic Average Return	Arithmetic Average Risk Premium	Arithmetic Average Unlevered Risk Premium	Smoothed Average Risk Premium	Average Debt/MVIC
1	38.99%	-0.41	57	0.87	17.00%	13.08%	6.17%	5.32%	4.98%	25.51%
2	29.61%	-0.53	58	0.82	17.51%	11.37%	6.40%	5.07%	5.07%	28.26%
3	25.17%	-0.60	51	0.85	17.43%	13.50%	6.58%	5.71%	6.02%	26.83%
4	22.02%	-0.66	58	0.94	16.98%	13.09%	6.10%	5.33%	7.12%	23.23%
5	19.69%	-0.71	52	0.98	18.47%	14.61%	7.70%	6.80%	7.04%	19.85%
6	17.89%	-0.75	53	1.06	17.88%	14.31%	7.40%	6.87%	7.00%	17.37%
7	16.85%	-0.77	47	1.10	19.22%	15.06%	8.10%	7.38%	8.12%	17.81%
8	15.79%	-0.80	52	1.11	20.07%	14.84%	7.94%	7.12%	8.36%	18.39%
Subject Company	14.60%	----->							?	
9	14.37%	-0.84	57	1.16	20.22%	16.59%	9.69%	8.78%	8.72%	19.37%
10	13.43%	-0.87	52	1.16	20.80%	15.78%	8.88%	7.93%	8.97%	20.21%
← // →										
24	3.78%	-1.42	81	1.29	26.33%	20.13%	13.22%	11.82%	13.71%	30.84%
25	2.18%	-1.66	113	1.29	28.90%	20.62%	13.72%	12.12%	13.78%	30.57%

Note: Some values intentionally blurred.

<sup>116</sup> The same three risk measures (for a hypothetical subject company) are used in all examples of estimating COE using the *Risk Study*, as outlined in Figure 41 on page 44.

<sup>117</sup> In addition to regression equations for interpolating risk premia between guideline portfolios in the *Risk Study*'s D exhibits, the *Size Study*'s A and B exhibits also provide regression equations for easy interpolation of risk premia between guideline portfolios, as do the C exhibits (for unlevered "A" exhibit risk premia).

<sup>118</sup> Please note that each exhibit has a different regression equation.

<sup>119</sup> The logarithmic relationship is base-10, and that the risk data is in percent, such that the log of 10 percent is log(10%), and not log(10). The formula to calculate a value's base-10 logarithm in Microsoft Excel is "=log(value)". Also note that the "\*" used in the regression equation is the symbol used in Microsoft Excel to denote the multiplication symbol, "x". The "\*" format is used to denote multiplication in the regression equations in the data exhibits.

# The Risk Study

Interpolate smoothed risk premium for each fundamental risk measure available for the subject company using the regression equations from the data exhibits. For example, the subject company in this example has a “coefficient of variation of operating margin” of 15.8 percent. The regression equation provided in Exhibit D-2 is:

$$\text{Smoothed Premium} = 13.505\% + 5.064\% * \text{Log}(\text{CV Op. Margin})$$

The interpolated smoothed risk premium is therefore 9.4 percent  $(13.505\% + 5.064\% * (-0.80))$ .

In the case of the third risk measure, the subject company has a “coefficient of variation of ROE” of 34.7 percent. The regression equation provided in Exhibit D-3 is:

$$\text{Smoothed Premium} = 10.632\% + 2.441\% * \text{Log}(\text{CV ROE})$$

The interpolated smoothed risk premium is therefore 9.5 percent  $(10.632\% + 2.441\% * (-0.46))$ .

After interpolating smoothed risk premia ( $RP_{m+u}$ ) for the subject company's available risk measures, Step 2 is complete.

**Step 3, Equity Risk Premium (ERP) Adjustment:** The *ERP Adjustment* is needed to account for any difference in the analyst's own ERP estimate and the historical (1963–2010) ERP. This example utilizes the *ERP Adjustment* (1.1%) established in Example 1a (page 31 in the full *Report*).

**Step 4, Estimate Cost of Equity (COE):** With the completion of Steps 1 through 3, the information needed to estimate a base cost of equity capital using the Buildup 3 method (using regression equations) is now completed. The risk premiums over the risk free rate ( $RP_{m+u}$ ) can be added to the risk-free rate ( $R_f$ ) and the *ERP Adjustment* to estimate an indicated cost of equity capital (COE) for the subject company, as illustrated in Figure 49:

The range of COE estimates for the hypothetical subject company in this example is 13.8 percent to 14.6 percent, with an average of 14.4 percent, and a median of 14.6 percent. The mean represents the average estimate, but the mean can be unduly influenced by very large or very small “outliers”. For this reason, the median estimate is generally preferred to the mean. The median estimate tends to not be as heavily influenced by very large or very small outliers, and can be considered a measure of the “typical” estimate in the group.

Use of fundamental accounting measures of risk allows for direct assessment of the riskiness of the subject company. For example, if the appropriate equity risk premium for the subject company when measuring risk by one or more fundamental risk measures is different than the equity risk premium based on size measures, this difference may be an indication of the “company-specific risk” of the subject company.<sup>120</sup>

**Figure 49: Buildup 3 COE Inputs (using regression equations)**

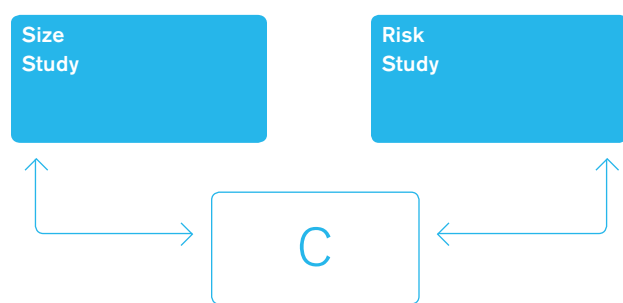
		Step 1		Step 2		Step 3		Step 4	
		Risk Free Rate, $R_f$		Smoothed Premium Over Risk Free Rate, $(RP_{m+u})$		<i>ERP Adjustment</i>		COE	
Risk Measure	Appropriate Exhibit								
Operating Margin	D-1	4.1%	+	8.6%	+	1.1%	=	13.8%	
Coefficient of Variation of Operating Margin	D-2	4.1%	+	9.4%	+	1.1%	=	14.6%	
Coefficient of Variation of ROE	D-3	4.1%	+	9.5%	+	1.1%	=	14.7%	
Mean (average) values		4.1%	+	9.2%	+	1.1%	=	14.4%	
Median (typical) values		4.1%	+	9.4%	+	1.1%	=	14.6%	

<sup>120</sup> *Valuing a Business*, 4th ed., Pratt et al, McGraw-Hill (2000), p 181. Company-specific risk factors can include concentration of customer base, key person dependence, key supplier dependence, or any number of other factors that are unique to the subject company.

# The Risk Study

## The C Exhibits Provide a “Link” Between the Size Study and the Risk Study

The C exhibits provide an important function—they serve as a “link” between the *Size Study* and the *Risk Study*. These exhibits can be used to compare a subject company’s fundamental risk characteristics to the *fundamental risk characteristics* of portfolios made up of similarly-sized companies. Specifically, the C exhibits can be used to compare the subject company’s fundamental risk characteristics to the fundamental risk characteristics of portfolios made up of companies of *similar size*.



Exhibits C-1 through C-8 also provide *unlevered* versions of the risk premia over the risk free rate found in the A exhibits ( $RP_{m+s}$ ). These unlevered premia ( $RP_{m+s, \text{unlevered}}$ ) are used in Examples 2a and 2b (see page 40 and 44, respectively, in the full *Report*) to estimate cost of equity capital assuming a firm is financed 100% with equity and 0% debt.<sup>126, 127</sup>

## Is Size Correlated with Market and Fundamental Risk Measures?

To calculate the statistics included in Exhibits C-1 through C-8, the fundamental risk characteristics are calculated for the same size-ranked portfolios that are created in the *Size Study*.<sup>128</sup> For example, Exhibit A-1 is comprised of 25 portfolios ranked by market value of equity. To calculate the fundamental risk characteristics found in Exhibit C-1, the three fundamental risk measures used to rank the portfolios in the *Risk Study* (five-year operating income margin, the coefficient of variation in operating income margin, and the coefficient of variation in return on book equity) are calculated for each of the 25 (market-value-of-equity-ranked) portfolios in Exhibit A-1.

These calculations are then made in the same fashion for each of the 25 size-ranked portfolios created for exhibits A-2 through A-8 (e.g. for each of the 25 portfolios ranked by “book value of equity” in Exhibit A-2, the three fundamental risk measures are calculated; then for each of the 25 portfolios ranked by “5-year average net income” in Exhibit A-3, the three fundamental risk measures are calculated, etc.).

It is important to understand that the 25 portfolios used to calculate the fundamental risk statistics included in the C exhibits are *different* from the 25 portfolios used to calculate the fundamental risk statistics included in the D exhibits. In the former case, the portfolios are ranked by each of eight alternative measures of size, and then the fundamental risk characteristics of each portfolio are calculated. In the latter case, the large base set of companies that the analyses of the *Report* begins with are ranked by each of the three fundamental risk measures to form 25 *risk-ranked* portfolios, and *then* the average risk characteristics of each portfolio are calculated. For example, if 10 companies were ranked by size, the order (from largest to smallest) may be quite different from the same 10 companies ranked by operating margin (from highest to lowest).

<sup>126</sup> The D exhibits also include “unlevered” risk premia, but these are unlevered versions of the corresponding “levered” risk premia found in the D exhibits. The unlevered risk premia in the C exhibits are unlevered versions of the corresponding “levered” risk premia found in the A exhibits.

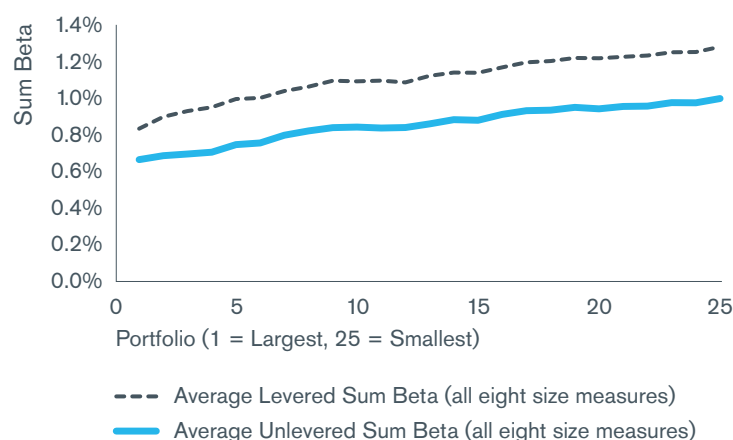
<sup>127</sup> The unlevered risk premia over the risk free rate found in the D exhibits ( $RP_{m+s, \text{unlevered}}$ ) are used in example 6 (see page 74 in the full *Report*) to estimate cost of equity capital using *Risk Study* inputs.

<sup>128</sup> Exhibits A-1 through A-8 and B-1 through B-8 use the *same* respective size-ranked portfolios, but calculate different statistics for each exhibit. For example, the 25 portfolios ranked by “book value of equity” are used in Exhibit A-2 and Exhibit B-2, but risk premia over the risk free rate ( $RP_{m+s}$ ) for use in a buildup method are calculated for Exhibit A-2, while risk premia over CAPM ( $RP_s$ , or “size premia”) for use in CAPM and Buildup 2 are calculated for Exhibit B-2.

# The Risk Study

The data suggests that size is correlated with market measures. For example, as size measures decrease in Graph 4 (from left to right), the beta (both levered and unlevered) of the portfolios increase (as expected).<sup>129</sup>

**Graph 4: Average Levered and Unlevered Sum Beta (all eight size measures) 1963–2010**



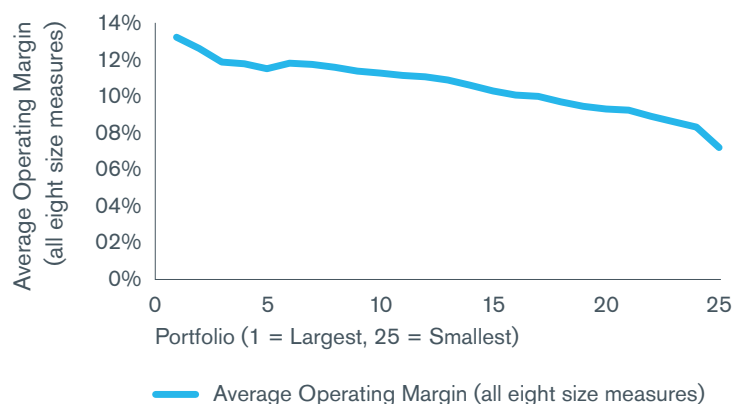
The data also suggests that this correlation extends to the three fundamental measures of risk. For example, in Graph 5a, as size measures decrease (from left to right), operating margin of the portfolios decreases (indicating increased risk), and in Graph 5b, as size measures decrease (from left to right), average coefficient of operating margin and average coefficient of variation of ROE of the portfolios increase (indicating increased risk).

While the correlation between fundamental measures of risk and size clearly demonstrated in Graph 5a and Graph 5b implies that there may be an embedded “size effect” component in the *Risk Study*’s  $RP_{m+u}$  premia, the magnitude of this embedded size effect is difficult to quantify. In any case, the size effect embedded in the *Risk Study*’s  $RP_{m+u}$  premia are in all likelihood not equivalent to the size effect embedded in the *Size Study*’s  $RP_{m+s}$  premia, which are a measure of risk in terms of the total effect of *market risk* and *size risk*.

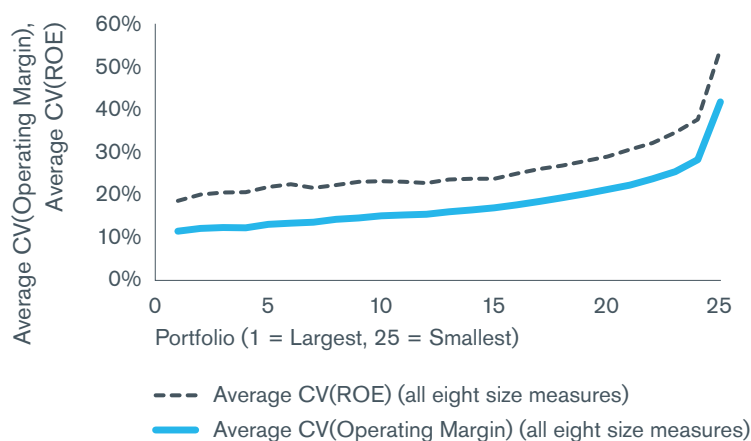
To avoid confusion between the two premia, and because the operating efficiencies (or lack thereof) of the subject company being captured by the use of accounting-based risk measures may offset the risk premium resulting from the size effect (or increase the risk

premium resulting from the size effect), the *Report* characterizes the *Risk Study*’s “risk premia over the risk free rate” ( $RP_{m+u}$ ) as being a measure of risk in terms of the total effect of *market risk* and *company-specific risk* (also known as “unsystematic risk”).

**Graph 5a: Average Operating Margin (all eight size measures) 1963–2010**



**Graph 5b: Average Coefficient of Operating Margin and Average Coefficient of Variation of ROE (all eight size measures)**



Generally, the three fundamental measures of risk display increasing risk as size decreases, as the historical unlevered risk premium increases and as the unlevered beta increases.<sup>130, 131</sup>

<sup>129</sup> In the research on “size” as reported in this report, we have determined that, in the context of the CAPM, the higher betas of the small companies explain some but not all of the higher average historical equity returns in these portfolios.

<sup>130</sup> Were one to calculate the respective correlations, those statistics would relate average portfolio statistics (e.g. average size vs. average risk) rather than correlation statistics across individual companies.

At the individual company level, the correlations are much lower.

<sup>131</sup> There are two notable exceptions to this pattern: Exhibit C-7 indicates that there is little differentiation in operating margin as size (as measured by sales) changes, and Exhibit C-8 indicates that there is little differentiation in operating margin as size (as measured by number of employees) changes. In both cases, however, the coefficient of variation of operating margin and the coefficient in variation of return on book equity indicate increasing risk as size (as measured by sales and number of employees) decreases, as in the other exhibits.

# The Risk Study

## The C Exhibits and Company-Specific Risk

The use of a portfolio's average historical rate of return to calculate a discount rate is based (in part) upon the implicit assumption that the risks of the subject company are quantitatively similar to the risks of the average company in the subject portfolio. If the risks of the subject company differ materially from the average company in the subject portfolio, then an appropriate discount rate may be lower (or higher) than a return derived from the average premium for a given portfolio. The data reported in exhibits C-1 through C-8 (where risk statistics are reported for each size category) may be helpful in making such a determination.

For example, assume that the size of the subject company based on the 5-year average net income equals \$20 million, placing it in the 23rd portfolio of exhibit A-3. The corresponding smoothed average risk premium over the risk free rate ( $RP_{m+s}$ ) from exhibit A-3 is 11.5 percent. The average operating margin for companies in the 23rd size-ranked portfolio equals 8.5% (see exhibit C-3, portfolio 23). If the subject company's operating margin is, say, 15.8%, it may be less risky than companies of similar size.

We can examine the relative operating margins to estimate an appropriate company-specific risk premium (positive or negative) that adjusts for the differences in the subject company and the typical company with a 5-year average net income equal to \$20 million. In exhibit D-1, for example, companies with average operating margins of approximately 15.8% make up the 8th (operating-margin-ranked) portfolio. The corresponding smoothed average risk premium over the risk free rate ( $RP_{m+u}$ ) from exhibit D-1 is 8.4 percent, which is significantly less than the 11.5 percent  $RP_{m+s}$  premia from exhibit A-3.

One can use the C exhibits to test each of the size measures to compare the fundamental risk characteristics of the subject company to the fundamental risk characteristics of portfolios made up of companies of similar size. If two or more (out of three) of the tests indicate that the subject company is less (or more) fundamentally risky than portfolios made up of companies of similar size, this can be a powerful argument in defending the downward (or upward) *direction* of a company-specific risk adjustment. Comparison of the subject company's fundamental risk characteristics to the corresponding risk premia found in the D exhibits can be helpful in estimating the *degree* of company-specific risk premium (positive or negative) that adjusts for the differences in the fundamental risk of the subject company and the average fundamental risk of companies comprising the portfolios in the D exhibits.

## Presentation of the Results

The C exhibits' 25 portfolios are ranked by the *same* eight alternative measures of size as the A and B exhibits, as described in Table 2 (see page 20).

Each of the exhibits C-1 through C-8 displays one line of data for each of the 25 size-ranked portfolios. The C exhibits include the statistics outlined in Table 3. In addition to information repeated from the A exhibits, the C exhibits report the additional datapoints for each of the 25 portfolios described in Table 6.

**Table 6: Statistics Reported for 25 Size-Ranked Portfolios in the C Exhibits (and not otherwise reported in the A Exhibits)**

<ul style="list-style-type: none"> <li>▪ Average debt to market value of equity.</li> </ul>	<ul style="list-style-type: none"> <li>▪ Operating Margin: The mean operating income for the prior five years divided by the mean sales for the prior five years. Operating income is defined as sales minus cost of goods sold plus selling, general, and administrative expenses plus depreciation.</li> </ul>
<ul style="list-style-type: none"> <li>▪ Arithmetic average historical unlevered risk premium over long-term Treasuries (average return on equity in excess of long-term Treasury bonds) since 1963. (<math>RP_{m+s, unlevered}</math>)</li> </ul>	<ul style="list-style-type: none"> <li>▪ Coefficient of Variation of Operating Margin: The standard deviation of operating margin over the prior five years divided by the average operating margin for the same years.</li> </ul>
<ul style="list-style-type: none"> <li>▪ "Smoothed" average historical unlevered risk premium: the fitted premium from a regression with the average historical unlevered risk premium as dependent variable and the logarithm of the average sorting criteria as independent variable (<math>RP_{m+s, unlevered}</math>)</li> </ul> <p>(The coefficients and constants from this regression analysis are in the top right hand corner of the exhibits)</p>	<ul style="list-style-type: none"> <li>▪ Coefficient of Variation of Return on Book Value of Equity: The standard deviation of return on book equity for the prior five years divided by the mean return on book equity for the same years. Return on book equity is defined as net income before extraordinary items minus preferred dividends divided by book value of common equity.</li> </ul>
<ul style="list-style-type: none"> <li>▪ Average unlevered beta calculated using the "sum beta" method applied to monthly returns for 1963 through the latest year (see the 2011 SBB/Valuation Yearbook pp. 77-78 for a description of the "sum beta" method).</li> </ul>	

# The High-Financial-Risk Study

The information and data in the *Duff & Phelps Risk Premium Report* (and in the online *Duff & Phelps Risk Premium Calculator*)<sup>132</sup> is primarily designed to be used to develop cost of equity capital (COE) estimates for the large majority of companies that are fundamentally healthy, and for which a “going concern” assumption is appropriate. A set of “high-financial-risk” companies is set aside and analyzed separately in the *High-Financial-Risk Study*.

The companies analyzed in the *High-Financial-Risk Study* are identified in a two-step process. First, companies that are losing money, have high leverage, or are in bankruptcy are identified and eliminated from the base set of companies used in the *Size Study* and *Risk Study*.<sup>133, 134</sup> It is possible to imagine companies that don’t have any of these characteristics but could still be classified as high-financial-risk (i.e. “distressed”), and it is also possible to imagine companies which do have one or more of these characteristics but are *not* distressed.

For this reason, these companies are further scrutinized in a second test where they are ranked by the appropriate Altman z-Score (for “manufacturing” companies or for “service” companies).<sup>135, 136</sup> Those companies identified as being in the “safe zone” (as defined by their z-Score) failed the first test, but passed the second test (z-Score), and are set aside and not used in any further analysis due to the inconclusive results. The remaining companies failed *both* the first test and the second test, and are placed in either the “gray” or “distressed” zone (as defined by their z-Score). The resulting base set of high-financial-risk companies is composed largely of companies whose financial condition is significantly inferior to the average, financially “healthy” public company.

The results of the *High-Financial-Risk Study* are presented in the H exhibits. The H exhibits provide risk premia that may be used in both buildup and CAPM estimates of cost of equity capital if the individual analyst has determined that the subject company is “high-financial-risk”.<sup>137</sup>

In cases in which the individual analyst has determined that the subject company is “high-financial-risk”, the high-financial-risk premia reported in the H exhibits should be used *instead of* the returns reported in the *Size Study*, and *not* added to those returns.

<sup>132</sup> The *Duff & Phelps Risk Premium Calculator* is available exclusively through Business Valuation Resources (BVR). For more information about the *Calculator*, visit: [www.bvresources.com/dp](http://www.bvresources.com/dp) or call toll free 1-(888)-287-8258.

<sup>133</sup> For a detailed discussion of how the high-financial-risk portfolios are created, see “High-Financial-Risk Study” in the portfolio methodology section on page 11.

<sup>134</sup> The number of companies eliminated in this screen varies from year to year. These companies represented up to 25% of the data set in recent years, but less than 5% in 1963. Certain technical changes in methodology have resulted in a greater number of companies falling into the high-financial-risk database than in versions of this study published prior to 2000.

<sup>135</sup> Altman z-Score is an accounting-data-based method designed to assess financial condition and developed originally for assessing the likelihood of bankruptcy. E. I. Altman, “Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy,” *The Journal of Finance*, Vol. 23, No. 4 (Sep., 1968), pp. 589-609; “Predicting Financial Distress of Companies: Revisiting the s-Score and Zeta Models,” July 2000; “Revisiting Credit Scoring Models in a Basel 2 Environment,” May 2002.

<sup>136</sup> Service industry companies are those SIC codes: 7200, 7300, 7500, 7600, 8000, 8100, 8200, 8300, 8400, 8700. Manufacturing are all other SIC codes, with the exception of SICs beginning with “6” (financial institutions) or “9” (government). SIC 6 and SIC 9 are not included in the *Report’s* analysis.

<sup>137</sup> The decision to apply a high-financial-risk premium is ultimately dependent on the analyst’s professional judgment, based upon the analyst’s detailed knowledge of the subject company.



# The High-Financial-Risk Study

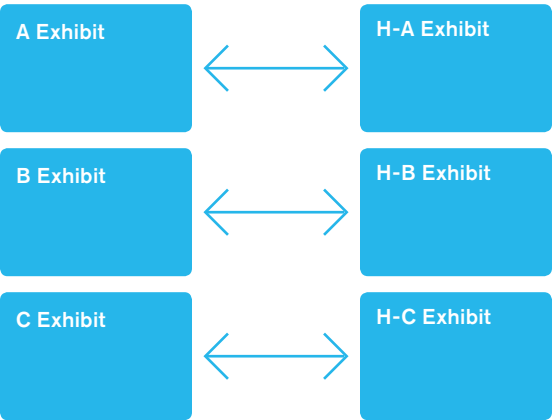
## The High-Financial-Risk “H” Exhibits

Exhibit H-A is the high-financial-risk equivalent of the A exhibits. “High-financial-risk premia over the risk free rate” for use in a buildup method are found in the H-A exhibits. These premia can be added to the risk free rate to estimate the cost of equity capital for a company that has been judged by the analyst to be high-financial-risk.

Exhibit H-B is the high-financial-risk equivalent of the B exhibits. “High-financial-risk premia over CAPM” (i.e. “size premia”) for use with the CAPM method are found in the H-B exhibits. These premia can be used in the CAPM to estimate the cost of equity capital for a company that has been judged by the analyst to be high-financial-risk.

Exhibit H-C is the high-financial-risk equivalent of the C exhibits. The H-C exhibits can be used to compare the subject company’s fundamental risk characteristics to the fundamental risk characteristics of portfolios made up of companies with similar z-Scores.

Figure 54: The A, B, and C Exhibits and Corresponding High-Financial-Risk Exhibits



Why isn't there an H-D exhibit? In the *Risk Study's D* exhibits, in addition to operating margin, two other measures of risk are examined (coefficient of variation in operating margin and coefficient of variation in return on equity). Because the denominators of these ratios are often negative for companies in the high-financial-risk portfolio as a result of either negative earnings or negative book value of equity, developing comparable “high-financial-risk” premia for these frequently results in meaningless statistics.



# The High-Financial-Risk Study

## Altman z-Score

Altman's z-Score was originally designed as a measure to predict the risk of failure up to two years prior to distress for a sample of manufacturing companies using financial data prepared according to the standards of the day. The accuracy of predicting the risk of failure diminished substantially as the lead time increased. The z-Score resulted from a statistical analysis of company data using the statistical technique of multiple discriminant analysis.

Altman has since offered improvements on the original z-Score, but the original z-Score is still frequently calculated as a convenient metric that captures within a single statistic a number of disparate financial ratios measuring liquidity, profitability, leverage, and asset turnover.<sup>138</sup>

Z-Score ratios are not strictly comparable across industries or across time (for instance, one would expect large differences in asset turnover among an industrial company or a retailer), and as such, are not used here as a predictor of bankruptcy per se, but as mechanism for ranking the high-financial-risk companies by their relative levels of distress.

The following z-Score model for publicly-traded "manufacturing" companies (i.e. excluding service industry companies) is used in preparing the analyses presented in the H-A, H-B, and H-C exhibits:

$$z = 1.2 x_1 + 1.4 x_2 + 3.3 x_3 + 0.6 x_4 + 0.999 x_5$$

where:

$z$  = Overall index

$x_1$  = Net working capital / total assets

$x_2$  = Retained earnings / total assets

$x_3$  = Earnings before interest and income taxes / total assets

$x_4$  = Market value of common equity / book value of total liabilities

$x_5$  = Sales / total assets

The companies are then sorted by z-Score into three portfolios:

- $z > 2.99$  = "safe zone"
- $1.80 < z < 2.99$  = "gray zone"
- $z < 1.80$  = "distress zone"

Companies in the "safe" zone (z-Score greater than 2.99) are set aside and not used in any further analysis. Companies in the "gray" zone (z-Score between 1.80 and 2.99) and companies in the "distressed" zone (z-Score less than 1.80) are used to form the portfolios from which the statistics presented in H-A, H-B, and H-C exhibits are calculated. Portfolios are rebalanced annually (i.e. the companies are re-ranked and sorted at the beginning of each year). Portfolio rates of return were calculated using an equal-weighted average of the companies in the portfolio.

<sup>138</sup> In applying any of the z-Score equations cited here, express the ratios in terms of their decimal equivalents (e.g.,  $x_1$  = working capital / total assets = 0.083).

# The High-Financial-Risk Study

The following z"-Score model for publicly-traded "service" industry high-financial-risk companies is used in preparing the analyses presented in the H-A, H-B, and H-C exhibits:

$$z'' = 6.56 x_1 + 3.26 x_2 + 6.72 x_3 + 1.05 x_4$$

where:

$z''$  = Overall index

$x_1$  = Net working capital / total assets

$x_2$  = Retained earnings / total assets

$x_3$  = Earnings before interest and income taxes / total assets

$x_4$  = Book value of common equity / book value of total liabilities

The companies are then sorted by z"-Score into three portfolios.

- $z'' > 2.60$  = "safe zone"
- $1.10 < z'' < 2.60$  = "gray zone"
- $z'' < 1.10$  = "distress zone"

Companies in the "safe" zone (z"-Score greater than 2.60) are set aside and not used in any further analysis. Companies in the "gray" zone (z"-Score between 1.10 and 2.59) and companies in the "distressed" zone (z"-Score less than 1.10) are used to form the portfolios from which the statistics presented in H-A, H-B, and H-C exhibits are calculated. Portfolios are rebalanced annually (i.e. the companies are re-ranked and sorted at the beginning of each year). Portfolio rates of return were calculated using an equal-weighted average of the companies in the portfolio.

Again, in both cases (manufacturing and service), we are not using the z-Score or z"-Score as a predictor of bankruptcy. Rather, companies are ranked in the *High-Financial-Risk Study* based on their relative levels of distress, using z-Score and z"-Score as proxies for "distress".

## Non-Public Companies and z'-Score

The traditional z-Score was developed using data for publicly traded companies, and one of the statistics utilizes stock price. This creates problems for application of the data to non-public companies. Altman developed a similar model using only the financial statement data for non-public companies. If the subject company is not publicly traded and not in the service industry, then the analyst can calculate a z-Score for non-public companies (the z'-Score) to compare with the data in the accompanying exhibits:

$$z' = 0.717 x_1 + 0.847 x_2 + 3.107 x_3 + 0.420 x_4 + 0.998 x_5$$

where:

$z'$  = Overall index

$x_1$  = Working capital / total assets

$x_2$  = Retained earnings / total assets

$x_3$  = Earnings before interest and income taxes / total assets

$x_4$  = Book value of common equity / book value of total liabilities

$x_5$  = Sales / total assets

The z'-Score's "zones of discrimination" loosely approximate the boundaries used to separate the z-Score and z"-Score ranked companies into portfolios, and are as follows:

- $z' > 2.90$  = "safe zone"
- $1.23 < z' < 2.90$  = "gray zone"
- $z' < 1.23$  = "distress zone"

While the H-A, H-B, and H-C exhibits are sorted by using the publically-traded company equations (z-Score for manufacturing companies and z"-Score for service companies) and are not strictly comparable to the z'-Score for non-public companies, the returns reported in these exhibits can be useful in developing cost of equity estimates based on the relative zones of discrimination.

# The High-Financial-Risk Study

## Measurement of Historical Risk Premiums

The high-financial-risk Study's H exhibits report average historical risk premiums for the period 1963 (the year that the *Compustat* database was inaugurated) through 2010. A long-run average historical risk premium is often used as an indicator of the expected risk premium of a typical equity investor. Returns are based on dividend income plus capital appreciation and represents returns after corporate taxes (but before owner level taxes).

To estimate historical risk premiums, an average rate of return is first calculated for each portfolio over the sample period. Portfolios with fewer than six companies in any given year are excluded in the averages. Lastly, the average income return earned on long-term Treasury bonds is subtracted from the portfolios' returns over the same period (using *S&P* data) to arrive at an average historical risk premium for investments in equity.

## Presentation of the Results

Each of the exhibits H-A, H-B, and H-C displays one line of data for each of the the z-Score- and z"-Score-ranked portfolios. These exhibits include the statistics outlined in Table 7.

For comparative purposes, the average returns from the *S&P* series for large companies (essentially the S&P 500 Index), small companies, and long-term government bond income returns for the period 1963 through the latest year are also reported on each exhibit.<sup>139</sup>

**Table 7: Statistics Reported for the z-Score- and z"-Score-ranked High-Financial-Risk Study's H-A, H-B, and H-C Exhibits**

Exhibit H-A	Exhibit H-B	Exhibit H-C
Beta calculated using the "sum beta" method applied to monthly returns for 1963 through the latest year (see the 2011 <i>S&amp;P Valuation Yearbook</i> pp. 77-78 for a description of the "sum beta" method).	Beta calculated using the "sum beta" method applied to monthly returns for 1963 through the latest year (see the 2011 <i>S&amp;P Valuation Yearbook</i> pp. 77-78 for a description of the "sum beta" method).	Arithmetic average historical risk premium over long-term Treasuries (average return on equity in excess of long-term Treasury bonds) since 1963 ( $RP_{m+s, high-financial-risk}$ ).
Standard deviation of annual historical equity returns.	Arithmetic average historical equity return since 1963.	Average carrying value of preferred stock plus long-term debt (including current portion) plus notes payable ("Debt") as a percent of MVIC since 1963.
Geometric average historical equity return since 1963.	Arithmetic average historical risk premium over long-term Treasuries (average return on equity in excess of long-term Treasury bonds) since 1963 ( $RP_{m+s, high-financial-risk}$ ).	Average debt to market value of equity.
Arithmetic average historical equity return since 1963.	Indicated CAPM premium, calculated as the beta of the portfolio multiplied by the average historical market risk premium since 1963 (measured as the difference between <i>S&amp;P</i> Large Stock total returns and <i>S&amp;P</i> income returns on long-term Treasury bonds).	Beta calculated using the "sum beta" method applied to monthly returns for 1963 through the latest year (see the 2011 <i>S&amp;P Valuation Yearbook</i> pp. 77-78 for a description of the "sum beta" method).
Arithmetic average historical risk premium over long-term Treasuries (average return on equity in excess of long-term Treasury bonds) since 1963 ( $RP_{m+s, high-financial-risk}$ ).	Premium over CAPM, calculated by subtracting the "Indicated CAPM Premium" from the "Arithmetic Risk Premium" ( $RP_{s, high-financial-risk}$ ).	Operating Margin: The mean operating income for the prior five years divided by the mean sales for the prior five years. Operating income is defined as sales minus cost of goods sold plus selling, general, and administrative expenses plus depreciation.
Average carrying value of preferred stock plus long-term debt (including current portion) plus notes payable ("Debt") as a percent of MVIC since 1963.		

<sup>139</sup> Source: Morningstar *EnCorr Analyzer* software.

# The High-Financial-Risk Study

## Overview of Methods Used to Estimate Cost of Equity Capital Using the High-Financial-Risk Study

The *High-Financial-Risk Study* provides two methods of estimating COE for a subject company that has been determined to be high-financial-risk: “Buildup 1-High-Financial-Risk” and “CAPM-High-Financial-Risk”. These methods are summarized in equation format, and summarized in Figure 55 in graphical “building blocks” format.

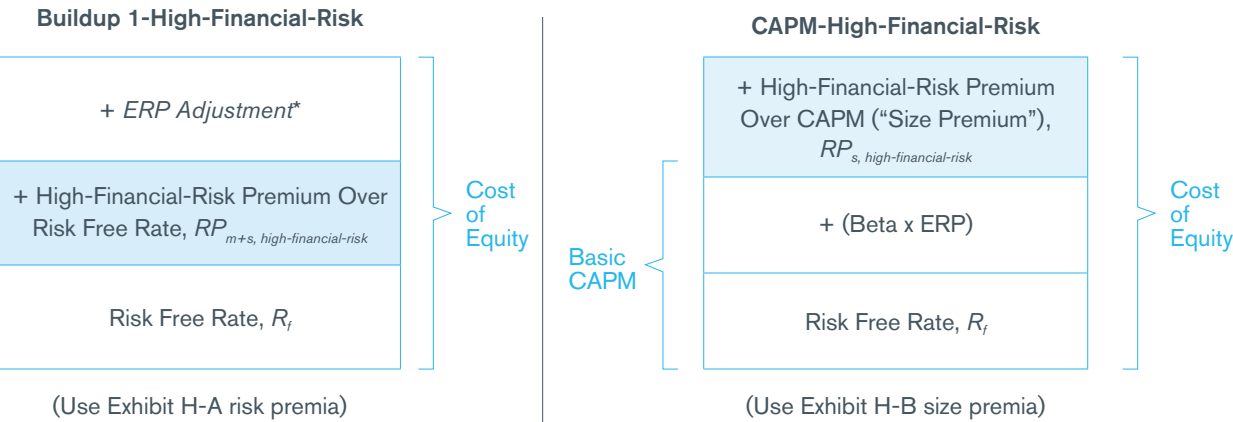
1) **Buildup 1-High-Financial-Risk**  
$$COE_{Buildup\ 1-High-Financial-Risk} = (Risk\ Free\ Rate) + (High\ Financial\ Risk\ Premium\ Over\ Risk\ Free\ Rate) + (Equity\ Risk\ Premium\ Adjustment)$$

Example 7: page 61

2) **Capital asset pricing model (CAPM)-High-Financial-Risk**  
$$COE_{CAPM-High-Financial-Risk} = (Risk\ Free\ Rate) + (Beta \times Equity\ Risk\ Premium) + (High-Financial-Risk\ Size\ Premium)$$

Example 8: page 89 (in the full *Report*)

Figure 55: Two Methods of Estimating Cost of Equity Capital with the *High-Financial-Risk Study*<sup>140</sup>



\* ERP Adjustment: The difference between the historical (1963–2010) equity risk premium (ERP) and a user of the *Duff & Phelps Report*’s own forward ERP estimate:

$$ERP\ Adjustment = User's\ ERP - Historical\ (1963-2010)\ ERP$$

The ERP Adjustment is made only in the “Buildup 1”, “Buildup 1-Unlevered”, “Buildup 1-High-Financial-Risk”, “Buildup 3”, and “Buildup 3-Unlevered” methods. Please refer to the individual examples provided for these models for more information.

NOTE: This section includes an example of using the *Report*’s risk premia data to estimate cost of equity capital using the “Buildup 1-High-Financial-Risk” method.

A complete example for using the *Report*’s risk premia to estimate cost of equity capital using the “CAPM-High-Financial-Risk” method is available in the full version of the 2011 *Report*.

<sup>140</sup> The relative sizes of the “building blocks” in Figure 55 do not necessarily represent the relative size of the various inputs. Also note that the names given to the models in the *Risk Premium Report* (e.g. “Buildup 1”, “Buildup 2”, “Buildup 3”, etc.) are naming conventions used within the *Report* to make referring to the different methods easier.

# The High-Financial-Risk Study

In this section, the information in Figure 56 will be used to estimate cost of equity capital for a hypothetical non-service (i.e. “manufacturing”) subject company.

**Figure 56: Subject Company Characteristics**  
(used in all examples)

	(in \$millions)		(in \$millions)
Market value of equity	\$80	Sales	\$250
Book value of equity	\$100	Current assets	\$75
Total assets	\$300	Current liabilities	\$50
Most recent year EBIT	-\$5	Retained earnings	\$75

The z-Score equation for a publicly-traded, non-service (i.e. “manufacturing”) subject company is:

$$z = 1.2 x_1 + 1.4 x_2 + 3.3 x_3 + 0.6 x_4 + 0.999 x_5$$

The inputs ( $x_1$ ,  $x_2$ ,  $x_3$ ,  $x_4$ , and  $x_5$ ) needed for the z-Score equation are calculated as shown in Figure 57:

Substituting these inputs into the z-Score equation yields a z-Score of 1.47:

$$z = 1.2(0.0833) + 1.4(0.2500) + 3.3(-0.0167) + 0.6(0.4000) + 0.999(0.8333)$$

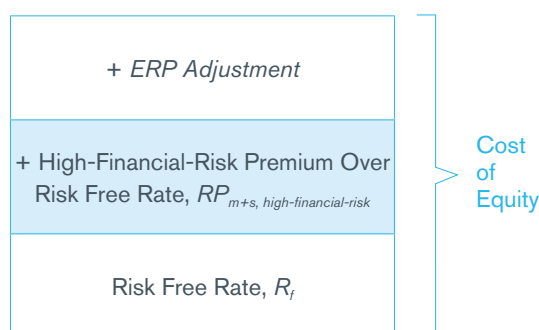
$$1.47 = 0.1000 + 0.3500 + (-0.0550) + 0.2400 + 0.8325$$

**Figure 57: z-Score Inputs Calculation**

$x_1$	=	Net working capital / total assets	=	(\$75 current assets - \$50 current liabilities) / (\$300 total assets)	=	0.0833
$x_2$	=	Retained earnings / total assets	=	(\$75 retained earnings) / (\$300 total assets)	=	0.2500
$x_3$	=	Earnings before interest and taxes / total assets	=	(-\$5 EBIT) / (\$300 total assets)	=	-0.0167
$x_4$	=	Market value of common equity / book value of total liabilities	=	(\$80 market value of equity) / (\$300 total assets - \$100 book value of equity)	=	0.4000
$x_5$	=	Sales / total assets	=	(\$250 sales) / (\$300 total assets)	=	0.8333

**Example 7: Estimating Cost of Equity Capital Using the “Buildup 1-High-Financial-Risk” Method**

## Buildup 1-High-Financial-Risk



(Use Exhibit H-A risk premia)

The buildup method is an additive model commonly used for calculating the required rate of return on equity. As the name implies, successive “building blocks” are summed, each representing the additional risk inherent to investing in alternative assets. An example of this is the extra return (i.e. “premium”), that investors demand for investing in stocks versus investing in a riskless security.<sup>141,142</sup>

This example utilizes the long-term risk free rate ( $R_f$ ) and the *ERP Adjustment* established in a previous example (the *Size Study*’s Buildup 1 method using “guideline portfolios”; see page 31 in the full *Report*). This mirrors the fact that for any given valuation engagement the same risk free rate and ERP will generally be used in each of the models presented by the individual analyst. Please note that for any given valuation engagement these inputs may (and probably will) be different than the ones used in the examples.

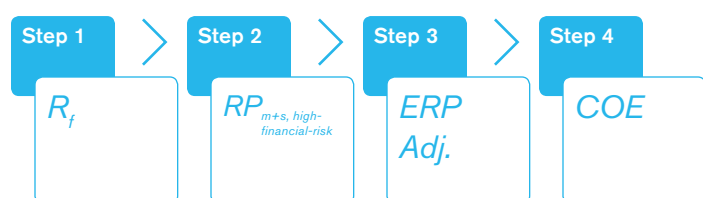
<sup>141</sup> Throughout this report the risk free asset ( $R_f$ ) is represented by the yield on a 20-year constant maturity Treasury Bond.

<sup>142</sup> For a detailed discussion of the buildup model, see “Estimating Cost of Equity Capital Using the ‘Buildup 1’ Method” on page 29 (in the full *Report*).

# The High-Financial-Risk Study

As in the Buildup 1 method, the “Buildup 1-High-Financial-Risk” method requires three pieces of information to estimate the cost of equity capital: a risk free rate ( $R_f$ ), a high-financial-risk premium over the risk free rate ( $RP_{m+s, \text{high-financial-risk}}$ ), and an *ERP Adjustment* (if necessary). All of the information needed is summarized in Figure 58.

**Figure 58: Information Needed to Estimate COE Using “Buildup 1-High-Financial-Risk”**



The *only* difference between estimating cost of equity capital (COE) using the Buildup 1 method and estimating COE using the Buildup 1-High-Financial-Risk method is that the “risk premium over the risk free rate” used in the latter method is a “high-financial-risk premium” ( $RP_{m+s, \text{high-financial-risk}}$ ), while the risk premia over the risk free rate used in the former are not.<sup>143</sup>

**Step 1 and Step 3:** Because the risk free rate in Step 1 ( $R_f$ , 4.1%) and the *ERP Adjustment* in Step 3 (1.1%) established in a previous example are being used in this example<sup>144</sup>, the only missing ingredient needed to estimate COE is the high-financial-risk premium over the risk free rate ( $RP_{m+s, \text{high-financial-risk}}$ ):

$$COE_{\text{Buildup 1-High-Financial-Risk}} = R_f + RP_{m+s, \text{high-financial-risk}} + \text{ERP Adjustment} =$$

$$COE_{\text{Buildup 1-High-Financial-Risk}} = 4.1\% + RP_{m+s, \text{high-financial-risk}} + 1.1\%$$

Determination of the high-financial-risk premium in Exhibit H-A for Step 2 is a three-step process (Steps 2a, 2b, and 2c):

**Step 2a:** Determine whether the characteristics of the subject company better match the characteristics of the companies included in Exhibits A-1 through A-8 (the 25 portfolios) or the characteristics of the high-financial-risk portfolios of companies as described above. The most straightforward way of doing this is to answer the following five questions about the subject company:<sup>145</sup>

- Is the subject company in bankruptcy or in liquidation?
- Is the subject company's “5-year average net income available to common equity” less than zero for the previous five years?
- Is the subject company's “5-year-average operating income” less than zero for the previous five years?
- Has the subject company had a negative book value of equity at any one of the company's previous five fiscal year-ends?
- Does the subject company have a debt-to-total capital ratio of more than 80%?

It is possible to imagine companies that don't have any of these characteristics, but could still be classified as high-financial-risk (i.e. “distressed”), and it is also possible to imagine companies which do have one or more of these characteristics but are *not* distressed.

If you answered “Yes” to one or more of the five questions, it *may* suggest that the subject company's characteristics are more like the companies that make up the “high-financial-risk” portfolios rather than like the “healthy” companies that make up the standard 25 portfolios, *but not necessarily so*. For example, a company may have a debt to total capital ratio greater than 80%, but this does not automatically imply that the company is in distress. A decision by the individual analyst that a company should be treated as “high-financial-risk” should be based on a detailed evaluation of the company's current financial condition and circumstances, and will generally involve more than a review of historical financial statistics and ratios. The decision to apply a high-financial-risk premium is ultimately dependent on the individual analyst's professional judgment and detailed knowledge of the subject company.<sup>146</sup>

<sup>143</sup> The “risk premia over the risk free rate” used in the Buildup 1 method are found in the A exhibits. A, B, C, and D risk premia are designed to be used to develop cost of equity capital (COE) estimates for the large majority of companies that are fundamentally healthy; the H exhibits are designed to be used to estimate COE for companies that the individual analyst has determined to be “high-financial-risk”.

<sup>144</sup> See the *Size Study*'s Buildup 1 method using “guideline portfolios” on page 31 (in the full *Report*).

<sup>145</sup> These five questions mirror the five criteria by which high-financial-risk companies are identified in (and eliminated from) the universe of US companies to form the base set of companies used in the *Size Study* and *Risk Study*.

<sup>146</sup> If the analyst determines that the subject company is not high-financial-risk, the returns reported in the exhibits in the *Risk Premium Report* for the 25 portfolios (the A, B, C, and D exhibits) may be more appropriate for the subject company than the returns reported in the H exhibits.

# The High-Financial-Risk Study

**Step 2b:** If the individual analyst determines that the subject company's characteristics better match the characteristics of the companies comprising the high-financial-risk portfolios, calculate the z-Score of the subject company using the appropriate z-Score equation:<sup>147</sup>

- z-Score is for publicly-traded, non-service, (i.e. "manufacturing") companies<sup>148</sup>
- z"-Score is for publicly-traded, "service" companies
- z'-Score is non-public, non-service companies.

**Step 2c:** Lastly, if the z-Score<sup>149</sup> of the subject company indicates that it is in the "gray zone" or "distress zone", match the z-Score of the subject company with the zone composed of companies with similar z-Scores in Exhibits H-A, and identify the corresponding average high-financial-risk premium over the risk free rate ( $RP_{m+s, \text{high-financial-risk}}$ ). For this example, the subject company is a manufacturing company with a z-Score of 1.47, placing it in the "distressed" portfolio (z-Scores < 1.8; see Figure 59). The corresponding high-financial-risk arithmetic average risk premium is 17.26 percent (17.3% rounded).

**Step 4:** Estimate a high-financial-risk cost of equity for the subject company by adding the average high-financial-risk premium over the risk free rate identified in Step 3 ( $RP_{m+s, \text{high-financial-risk}}$ ) to the risk free rate  $R_f$  and the *ERP Adjustment* (if appropriate).

$$COE_{\text{Buildup 1-High-Financial-Risk}} = R_f + RP_{m+s, \text{high-financial-risk}} + ERP \text{ Adjustment} = 22.5\% = 4.1\% + 17.3\% + 1.1\%$$

The "high-financial-risk" COE estimate for the hypothetical subject company in this example is 22.5 percent.

**Figure 59: "Buildup 1-High-Financial-Risk" COE Input**  
**Exhibit H-A, High-Financial-Risk Premia Over the Risk Free Rate**

## Companies Ranked by z-Score

Historical Equity Risk Premium: Average Since 1963

High-Financial-Risk Company Data for Year Ending December 31, 2010

Portfolio Rank	Beta (SumBeta) Since '63	Standard Deviation of Returns	Geometric Average Return	Arithmetic Average Return	Arithmetic Average Risk Premium	Average Debt/MVIC
<b>Manufacturing (z-Score)</b>						
1.8 to 2.99 (gray zone)	1.58	37.20%	15.05%	21.50%	14.80%	47.04%
< 1.8 (distress zone)	1.65	40.01%	16.66%	24.16%	17.26%	58.28%
<b>Service Industry (z"-Score)</b>						
1.1 to 2.59 (gray zone)	1.63	44.33%	14.08%	28.00%	21.10%	42.31%
< 1.1 (distress zone)	1.71	46.48%	19.93%	35.40%	28.40%	50.33%

NOTE: Some values intentionally blurred.

<sup>147</sup> In all examples here, the z-Score for publicly-traded, non-service (i.e. "manufacturing") companies is used.

<sup>148</sup> While the H-A, H-B, and H-C exhibits are ranked by z-Score and z"-Score and are not strictly comparable to the z'-Score for non-public companies, the returns reported in these exhibits can be useful in developing cost of equity estimates based on the relative zones of discrimination.

<sup>149</sup> Or, as appropriate, z"-Score or z'-Score.

# The High-Financial-Risk Study

## Additional information on Company-Specific Risk

The *Risk Study* provides analysis that correlates historical equity returns (and historical risk premiums) directly with three measures of company-specific risk derived from accounting information (five-year operating income margin, the coefficient of variation in operating income margin, and the coefficient of variation in return on book equity). These may also be called “fundamental” measures of company risk to distinguish them from stock market-based measures of equity risk (e.g. beta). The *Risk Study* demonstrates that as company size decreases, measures of risk calculated from financial statement data do, as a matter of fact, tend to increase.

In the *High-Financial-Risk Study*, one measure of accounting-data-based fundamental risk (five-year operating income margin) was examined for portfolios formed by ranking public companies by z-Score (manufacturing companies) and z”-Score (service companies).<sup>155, 156</sup>

The H-C exhibits can be used to compare a subject company's operating margin to the operating margins of portfolios made up of companies with similar z-Scores (or z”-Scores). For example, in the previous examples (Example 7 and Example 8), the subject company was a manufacturing company with a z-Score of 1.47, placing it in the “distressed” zone in exhibits H-A and H-B. The average operating margin (2.5%) of the companies comprising the portfolio used to calculate the statistics for “manufacturing” companies in the distress zone in exhibits H-A and H-B is published in Exhibit H-C (see Figure 62).

If the hypothetical subject company in these examples has a higher operating margin of, say 7 percent, it may be less risky than companies with similar z-Scores. This may suggest that a downward company-specific risk adjustment is justified.

Figure 62: Exhibit H-C

### Companies Ranked by Market Value of Equity: Comparative Risk Characteristics

High-Financial-Risk Company Data for Year Ending December 31, 2010

Portfolio Rank	Arithmetic Average Risk Premium	Average Debt to MVIC	Average Debt to Market Value of Equity	Beta (SumBeta) Since '63	Average Operating Margin
<b>Manufacturing (z-Score)</b>					
1.8 to 2.99 ( <i>gray zone</i> )	14.6%	47.04%	88.8%	1.58	3.1%
< 1.8 ( <i>distress zone</i> )	17.3%	58.28%	139.7%	1.65	2.5%
<b>Service Industry (z”-Score)</b>					
1.1 to 2.59 ( <i>gray zone</i> )	21.1%	42.31%	73.3%	1.63	3.4%
< 1.1 ( <i>distress zone</i> )	28.5%	50.33%	101.3%	1.71	2.2%

NOTE: Some values intentionally blurred.

<sup>155</sup> Because the denominators of the other two ratios (coefficient of variation in operating income margin, and coefficient of variation in return on book equity) are often negative for companies in the high-financial-risk portfolios (as a result of either negative earnings or negative book value of equity), developing comparable “high-financial-risk” premia for them frequently results in meaningless statistics.

<sup>156</sup> Operating margin is defined here as the mean operating income for the prior five years divided by the mean sales for the prior five years. Operating income is defined as sales minus cost of goods sold plus selling, general, and administrative expenses plus depreciation.



# The High-Financial-Risk Study

Table 8 provides additional summary statistics for the H exhibits' z-Score- and z"-Score-ranked portfolios.<sup>157</sup> For example, portfolios made up of manufacturing companies with an average z-Score less than 1.8 had an average book value of equity of \$ 93.788 million and an average 5-year average net income of -\$16.540 million.

**Table 8: Companies Ranked by Sorting Criteria**  
High-Financial-Risk Company Data for Year Ending December 31, 2010  
Portfolio Details (\$mils.)

Portfolio Rank	Number Number as of 2010	Portfolio Average							
		Market Value of Equity	Book Value of Equity	5-Year Average Net Income	Market Value of Invested Capital	Total Assets	5-Year Average EBITDA	Sales	Number of Employees
Manufacturing (z-Score)									
1.8 to 2.99 (gray zone)	123	168.933	130.179	(5.690)	338.415	487.036	56.662	752.698	2,600
< 1.8 (distress zone)	277	205.947	93.788	(16.540)	609.226	613.120	61.662	474.954	1,606
Service Industry (z"-Score)									
1.1 to 2.59 (gray zone)	29	210.376	152.083	(1.326)	324.831	361.914	26.870	249.228	1,189
< 1.1 (distress zone)	65	118.710	35.311	(11.006)	189.694	184.182	16.294	198.233	1,064

<sup>157</sup> The information in Table 8 was published as "Exhibit H-E" in previous Reports.

# Data Exhibits

NOTE: The data exhibits are not included in this document (the data exhibits are available in the complete *2011 Report*).

# New in 2011: The Duff & Phelps Risk Premium Calculator (web-based)

In 2011 we introduce the web-based *Duff & Phelps Risk Premium Calculator*. The *Calculator* automatically estimates levered and unlevered cost of equity capital (COE) for your subject company dependent on its size and risk characteristics (for any valuation date from January 1, 1996 to present), using both the capital asset pricing model (CAPM) and buildup models.

The *Calculator* is easy to use, saves time, and automatically provides full summary output in both Microsoft Word and Microsoft Excel format. In addition, the *Calculator* automatically looks up the long-term risk free rate for your valuation date<sup>1</sup>, automatically makes the important (but often overlooked) “ERP Adjustment” to your subject company’s COE estimates, and automatically adjusts an *SBB* industry risk premium (IRP) so that it can be used in a Buildup model using *Risk Premium Report* size premia.<sup>2</sup>

## Calculator Features

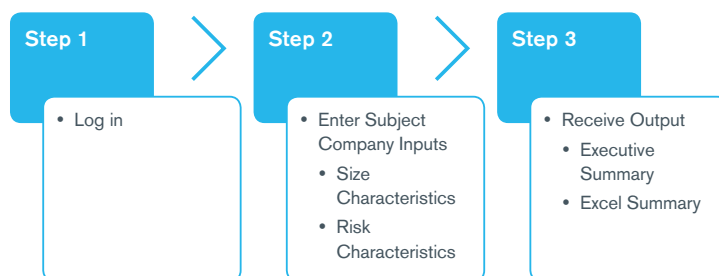
- Anytime, anywhere access at [www.bvmarketdata.com/DP.RPC](http://www.bvmarketdata.com/DP.RPC)
- Complete historical database of risk premia and size premia data (1996–2011)
- Automatic output
  - Executive Summary of COE estimates, including CAPM, Buildup, and unlevered COE
  - Microsoft Excel output of all underlying values and calculations
- Easy to use / Saves time

The *Calculator* employs the methodology and data published in the *Duff & Phelps Risk Premium Report*, which has provided financial and valuation professionals defensible cost of capital data and methodology since 1996<sup>3</sup>

## Calculator Tour

Duff & Phelps designed the *Calculator* with two simple goals: the user experience had to be as easy and smooth as possible, and the *Calculator* had to maintain the same analytical horsepower, data, and methodology “under the hood” as is found in the *Risk Premium Report*.

There are three simple steps needed to calculate cost of equity capital (COE) using the *Calculator*.



<sup>1</sup> 20-year constant maturity Treasury bond yield as of your valuation date. Source: The Board of Governors of the Federal Reserve System.

<sup>2</sup> Duff & Phelps does not publish IRPs. A source of IRPs is Morningstar’s *Ibbotson SBB Valuation Yearbook*, (Chicago, Morningstar), Chapter 3, “The Buildup Method”, Table 3-5.

<sup>3</sup> For detailed information about the *Size Study*, *Risk Study*, and *High-Financial-Risk Study* included in the *Risk Premium Report* (and now available in the *Risk Premium Calculator*), please see pages 18, 39, and 55, respectively.

# New in 2011: The Duff & Phelps Risk Premium Calculator (web-based)

**Step 1:** Log in at [www.bvmarketdata.com/DP.RPC](http://www.bvmarketdata.com/DP.RPC)

Image 1 – Logging in

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Risk Premium Calculator

Please sign in using your User ID and Password. Forgot password?

User ID:  
JoeAnalyst@Valuations.com

Password:  
•••••

login

Calculator:  
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**Step 2a:** Enter your subject company's name, and the valuation date.

Image 2 – Subject Company Name and Valuation Date

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Risk Premium Calculator

Use the fields below to enter information for your calculation.

Subject Company: ABC Widgets

Valuation Date: 12/31/2010

Choose the Report year you would like to use for your calculations.

Duff & Phelps Risk Premium Report: 2011 Report (data through Dec. 31, 2010)

Is the Subject Company a financial services company (SIC Code 6)? No

Financial service companies are excluded from the analysis used to develop the premiums used in the Duff & Phelps Risk Premium Online Calculator, and therefore using the results in the Duff & Phelps Risk Premium Online Calculator (or the accompanying Duff & Phelps Risk Premium Report) to estimate the cost of capital for a financial services company is not recommended.

☐ I understand and wish to override the financial services restriction.

Please note that overriding the financial services restriction is not recommended.

reset

next

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## New in 2011: The Duff & Phelps Risk Premium Calculator (web-based)

**Step 2b:** An optional set of questions and inputs is provided if the individual analyst has determined that the subject company is “high-financial-risk”.<sup>4</sup>

**Image 3 – Optional “High-Financial-Risk” Information**

### Risk Premium Calculator

Select whether your Subject Company is high financial risk.

Subject Company: ABC Widgets

Valuation Date: 12/31/2010

Is Subject Company high financial risk (i.e. "distressed")?

**High Financial Risk**

Is the Subject Company in bankruptcy or in liquidation?

Is the company's 5-year-average net income available to Common Equity for the previous five years less than zero?

Is the company's 5-year-average Operating Income for the previous five years less than zero?

Has the company had a negative Book Value of Equity at any of the previous five fiscal year-ends?

Is the company's Debt-To-Total Capital ratio greater than 80%?

**Altman z-Score** (in millions)

Subject Company type

Working Capital

Total assets

Retained earnings

Earnings before interest and income taxes (EBIT)

Market value of common equity

Book value of total liabilities

Sales

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The five questions in this step mirror the five criteria by which high-financial-risk companies are identified in (and eliminated from) the universe of US companies to form the base set of companies used in the *Size Study* and *Risk Study*.

If you answer “Yes” to one or more of the five questions, it *may* suggest that the subject company's characteristics are more like the companies that make up the “high risk” portfolios rather than like the “healthy” companies that make up the standard 25 portfolios, but *not necessarily* so. For example, a company may have a debt to total capital ratio greater than 80%, but this does not automatically imply that the company is in distress.

<sup>4</sup> The information and data in the *Duff & Phelps Risk Premium Calculator* is primarily designed to be used to develop cost of equity capital (COE) estimates for the large majority of companies that are fundamentally healthy, and for which a “going concern” assumption is appropriate. A set of “high-financial-risk” companies is set aside and analyzed separately in the *High-Financial-Risk Study*. The decision to apply a high-financial-risk premium is ultimately dependent on the analyst's professional judgment, based upon the analyst's detailed knowledge of the subject company. Please note that *High-Financial-Risk Study* output is available for calendar year 2010 valuation dates (and later) only.

**Image 4 – Basic Inputs Screen (not filled out)**

Also note that the *Calculator* provides information and tips which appear if you hover your mouse cursor over one of the information icons . These helpful tips provide quick assistance if you need the definition of an input, or the source of an input.

| 70

## New in 2011: The Duff & Phelps Risk Premium Calculator (web-based)

Fill in your subject company's size characteristics and risk characteristics, as shown in Image 5.

Image 5 – Basic Inputs Screen (filled out)

### Risk Premium Calculator

Use the fields below to enter information for your calculation.

Subject Company: ABC Widgets  
Valuation Date: 12/31/2010

**General Inputs**

Equity Risk Premium: 5.50%

Beta: 1.20

Industry Risk Premium: 1.50%

Risk Free Rate: 4.13%

**Size Study Inputs** (in millions except number of employees)

Market Value of Common Equity: \$120.00

Book Value of Equity: \$100.00

5-Year Average Net Income: \$10.00

Market Value of Invested Capital: \$180.00

Total Assets: \$300.00

5-Year Average EBITDA: \$30.00

Net Sales: \$700.00

Number of Employees: 200

**Risk Study Inputs** (in millions)

Most recent fiscal year relative to the valuation date	2010	2009	2008	2007	2006
Net Sales	\$700.00	\$800.00	\$850.00	\$750.00	\$900.00
Operating Income	\$150.00	\$120.00	\$130.00	\$80.00	\$140.00
Book Value of Equity	\$820.00	\$710.00	\$630.00	\$540.00	\$500.00
Net Income Before Extraordinary Items	\$110.00	\$80.00	\$90.00	\$40.00	\$100.00

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Under "General Inputs", enter the equity risk premium (ERP) you want used in all cost of equity capital (COE) calculations. For example, many users of the *Risk Premium Report* use the *Duff & Phelps Recommended ERP*, which was 5.5 percent at the end of 2010.<sup>6,7,8</sup>

Also under "General Inputs", enter a beta if you would like COE estimated using the capital asset pricing (CAPM) model, and an industry risk premium (IRP) from the *SBBI Yearbook* if you would like COE estimated using a buildup model that utilizes an IRP to account for market risk.

Only one (of the eight total) *Size Study* inputs is required, but enter as many of the eight values as possible for best results.

If you wish to receive cost of equity capital estimates derived using the *Risk Study*, the three most recent years of information are required (for best results, enter the most recent five years of information).

Please note that the Calculator automatically makes the important (but often overlooked) "ERP Adjustment" to your subject company's COE estimates, and automatically adjusts an SBBI industry risk premium (IRP) so that it can be used in a Buildup model using *Risk Premium Report* size premia.

<sup>6</sup> For more information on the equity risk premium, see *Cost of Capital: Applications and Examples* 4th ed., by Shannon P. Pratt and Roger J. Grabowski (John Wiley & Sons, Inc., 2010), Chapter 9, "Equity Risk Premium", pages 115–158.

<sup>7</sup> See Roger J. Grabowski, "Developing the Cost of Equity Capital: Risk-Free Rate and ERP During Periods of 'Flight to Quality'". This paper will appear in the *Business Valuation Review* and can also be downloaded at Duff & Phelps' Cost of Capital site at [www.duffandphelps.com/CostofCapital](http://www.duffandphelps.com/CostofCapital)

<sup>8</sup> If no ERP is entered, the historical ERP as calculated over the time horizon 1963 to the (year of the valuation date -1) is used. For example, for a calendar year 2011 valuation date, if no ERP is entered by the analyst in "General Inputs" the ERP as calculated from 1963–2010 (4.4%) would be used in all calculations; for a calendar year 2010 valuation date, if no ERP is entered by the analyst in "General Inputs", the ERP as calculated from 1963–2009 (4.3%) would be used in all calculations, etc.

## New in 2011: The Duff & Phelps Risk Premium Calculator (web-based)

Prior to calculating COE estimates for your subject company, the *Calculator* displays a summary of all of your inputs as shown in Image 6. At this point you can review your inputs, and change them (if necessary).

By clicking the “Confirm” button, you are agreeing that all of your inputs are as you intend, and the *Calculator* then calculates cost of equity capital (COE) estimates for your subject company.

Image 6 – Confirm / Change Inputs

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Risk Premium Calculator

Please verify the information below and make any necessary changes.

Subject Company: ABC Widgets  
Valuation Date: 12/31/2010  
For your valuation date (December 31, 2010), the default report is the 2010 Duff & Phelps Risk Premium Report (data through December 31, 2009). You have chosen to use the 2011 Duff & Phelps Risk Premium Report (data through December 31, 2010).  
You have indicated that the Subject Company, ABC Widgets, is NOT a financial services company (finance, insurance, or real estate). The Duff & Phelps Risk Premium Online Calculator is appropriately used to estimate cost of equity for non-financial firms, so it is OK TO PROCEED with cost of capital estimation for your Subject Company.

**High Financial Risk**  
You have indicated that your subject company is not "high financial risk".

**General Inputs**

- You entered an Equity Risk Premium (ERP) of 5.5% to use in all cost of equity estimates.
- You entered a beta ( $\beta$ ) of 1.20 to use in Capital Asset Pricing Model (CAPM) cost of equity estimates.
- You entered an IRP of 1.5%. An adjusted IRP of 1.2% will be used in the 'Buildup 2' cost of equity estimate. The adjusted IRP is calculated by: (IRP entered by the USER) x (the ERP entered by the USER) / (historical ERP (1926-2010)).

**Size Study Inputs**

- You entered a MARKET VALUE OF EQUITY of \$120.00 million
- You entered a BOOK VALUE OF EQUITY of \$100.00 million
- You entered a 5-YEAR AVERAGE NET INCOME of \$10.00 million
- You entered a MARKET VALUE OF INVESTED CAPITAL (MVIC) of \$180.00 million
- You entered TOTAL ASSETS of \$300.00 million
- You entered 5-YEAR AVERAGE EBITDA of \$30.00 million
- You entered a SALES of \$700.00 million
- You entered 200 as the NUMBER OF EMPLOYEES

**Risk Study Inputs**

- You have entered enough information to calculate COE based upon the Risk Study.

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# New in 2011: The Duff & Phelps Risk Premium Calculator (web-based)

After the *Calculator* calculates estimates of the subject company's cost of equity capital (COE), an abbreviated online "results preview" is displayed, as shown in Image 7.

Image 7 – Cost of Equity Capital (COE) Estimates (online "results preview")

## Risk Premium Calculator

Below are the summary results of your calculation, as well as links to download the details and an executive summary.

**Download Results**

Support and Detail workbook: [XLSX](#)

Executive Summary document: [DOCX](#)

You may also receive your calculation results via [email](#).

[back](#) [new calculation](#)

Click the "DOCX" and "XLSX" links for instant download of *Executive Summary* and *Support and Detail* documents.

Cost of Equity Capital Estimates – Summary of all Size Study models  
Models: Buildup 1, Buildup 2, CAPM, and Unlevered COE Estimates

Subject Company: ABC Widgets  
Valuation Date: 12/31/2010

Beta: a beta ( $\beta$ ) of 1.20 has been entered by the USER to use in Capital Asset Pricing Model (CAPM) calculations of COE estimates.

1.2

Source of risk premia information: 2011 Duff & Phelps Risk Premium Report

Equity Risk Premium (ERP) entered by USER and used in all calculations: 5.5%

Historical Equity Risk Premium (ERP) 1963–2010: 4.4%

ERP adjustment: an ERP adjustment of 1.1% is added to Buildup 1, Risk Study (Buildup 3), and Unlevered Risk premia to adjust for the difference in the historical ERP (1963–2010) and the ERP entered by the USER, as calculated below: 5.5% - 4.4% = 1.1%

Long-term risk free rate as of December 31, 2010 (used in all calculations): 4.1%

Adjusted Industry Risk Premium (IRP): an adjusted IRP of 1.2% is used in the 'Buildup 2' cost of equity estimate. The adjusted IRP is calculated by: (IRP entered by the USER) x (the ERP entered by the USER) / (historical ERP (1926–2010)), as shown below: 1.5% x (5.5% / 6.7%) = 1.2%

Summary of Cost of Equity (COE) Estimates

Buildup 1 COE Estimates			CAPM COE Estimates		
	Mean	Median		Mean	Median
Guideline Company Method	17.6%	17.5%	Guideline Company Method	17.3%	17.1%
Regression Equation Method	17.4%	17.5%	Regression Equation Method	17.2%	17.2%

Your complete (as opposed to online "results preview") COE estimate report includes an "Executive Summary" in Microsoft Word format and a "Support and Detail" Microsoft Excel workbook, which can be instantly downloaded by clicking on the "XLSX" and "DOCX" links at the top of the online "results preview" page, as indicated in Image 7.

# New in 2011: The Duff & Phelps Risk Premium Calculator (web-based)

Your complete COE estimate report includes:

## Executive Summary (in Microsoft Word format)

The *Executive Summary* is a high-level overview of data sourcing information, key inputs used in calculations, and cost of equity capital (COE) estimates for all models employed (with your subject company's information plugged into each model's equation)<sup>9</sup>, plus a concluded range of COE estimates for your subject company (using both the *Size Study* and *Risk Study*).<sup>10</sup>

Because the *Executive Summary* is in Microsoft Word format, you can edit it and format it to suit your individual needs. For example, inserting your own disclaimer information or adding your company logo is easy.

## Support and Detail summary of all inputs and calculations (in Microsoft Excel format)

The *Support and Detail* workbook includes a summary of your subject company's size and fundamental risk characteristics (and all other inputs), and complete documentation of calculations and inputs for each of the models used to estimate cost of equity capital (COE) for your subject company.

The *Support and Detail* workbook also includes the data exhibits<sup>11</sup> for each of the guideline portfolios that match your subject company (by size and/or fundamental risk). This important information includes a complete listing of size premia and risk premia (both levered and unlevered), average arithmetic and geometric returns, sum betas, average debt to MVIC, average debt to market value, average operating margin, average coefficient of variation of operating margin, average coefficient of variation of ROE, z-Score, and more.

An additional (and very important) capability of the *Calculator* that is documented in the *Support and Detail* workbook is that the *Calculator* automatically maps your subject company's size measures from the *Size Study* to portfolios of companies sorted by the three fundamental risk measures analyzed in the *Risk Study*, and then analyzes whether an upward or downward "company-specific" risk adjustment is indicated for each of the three fundamental risk factors. Why is this important? If two or more of the indicators are saying the same thing (upward adjustment or downward adjustment), it is a very powerful argument in defending a company-specific risk adjustment.

An additional (and very important) capability of the *Calculator* that is documented in the *Support and Detail* workbook is that the *Calculator* automatically maps your subject company's size measures from the *Size Study* to portfolios of companies sorted by the three fundamental risk measures analyzed in the *Risk Study*, and then analyzes whether an upward or downward "company-specific" risk adjustment is indicated for each of the three fundamental risk factors. Why is this important? If two or more of the indicators are saying the same thing (upward adjustment or downward adjustment), it is a very powerful argument in defending a company-specific risk adjustment.

Because the *Support and Detail* workbook is in Microsoft Excel format, you can edit it and format it to suit your individual needs. The workbook also includes a table of content tab and section divider tabs, so that when printed it is an organized, polished document ready for insertion into your valuation engagement report as a detailed "support, sourcing, and documentation" section designed to accompany the *Executive Summary*.

For free samples of complete *Executive Summary* and *Support and Detail* outputs, or for more information about the *Calculator*, please visit:

[www.BVResources.com/dp](http://www.BVResources.com/dp)

<sup>9</sup> Please note that the number of models employed is dependent on the completeness (or lack thereof) of subject company inputs entered by *Calculator* users.

<sup>10</sup> The *Duff & Phelps Risk Premium Calculator* is based upon the *Duff & Phelps Risk Premium Report*. The *Risk Premium Report* includes a *Size Study* (which analyzes the relationship between equity returns and company size using up to eight measures of company size), a *Risk Study* (which analyzes the relationship between equity returns and accounting-based fundamental risk measures), and a *High-Financial-Risk Study* (which analyzes the relationship between equity returns and high-financial-risk, as measured by the Altman z-Score).

<sup>11</sup> Exhibits A-1 through A-8 (used in the Buildup method); B-1 through B-8 (used in the CAPM method); C-1 through C-8 (used to compare your subject company's risk characteristics to portfolios comprised of companies of the same size as the subject company); D-1 through D-8 (used to estimate COE based upon fundamental risk factors); H-A, H-B, and H-C ("high-financial-risk" premia and size premia), and z-Score calculations. Please note that the data and information included in the *Support and Detail* workbook is dependent on the completeness (or lack thereof) of subject company inputs entered by *Calculator* users.

## New in 2011: The Duff & Phelps Risk Premium Calculator (web-based)

### Product Purchasing Information

You can purchase the *Duff and Phelps Risk Premium Calculator* exclusively through Business Valuation Resources (BVR) at:

[www.bvresources.com/dp](http://www.bvresources.com/dp)

1-(888)-287-8258.

All purchases of the *Duff & Phelps Risk Premium Calculator* include a copy of the *Duff & Phelps Risk Premium Report*.

### Calculator Option 1

**Includes 16 years of size premia and risk premia data (1996 – 2011):**

1-year subscription includes a copy of the *2011 Duff & Phelps Risk Premium Report* and unlimited access to *Duff & Phelps Risk Premium Calculator* data from 1996-2011. Estimate cost of equity capital for any valuation date from January 1, 1996 to present. \$759

### Calculator Option 2

**Includes 2 years of size premia and risk premia data (2010 – 2011):**

1-year subscription includes a copy of the *2011 Duff & Phelps Risk Premium Report* and unlimited access to the most recent two years of *Calculator* data (currently 2010-2011). Estimate cost of equity capital for any valuation date from January 1, 2010 to present. \$399

### Calculator Option 3

**Single Year Duff & Phelps Risk Premium Report:** Includes 1-time use of *Risk Premium Calculator*. \$275

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For more information, visit:  
[www.duffandphelps.com](http://www.duffandphelps.com)

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