# Problems with Cost of Capital Estimation in the Current Environment - Update ${ }^{1}$ 

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## Executive Summary

The current economic environment has created challenges in estimating the cost of equity capital ("COEC") and in estimating the appropriate overall cost of capital (i.e., the weighted average cost of capital or "WACC"). Since October 2008, new complications have arisen in estimating the cost of capital. Traditional methods typically employed in estimating the COEC and the WACC are subject to significant estimation and data input problems. This article attempts to address some of these issues and offer some specific recommendations on dealing with these issues.

First, U.S. Treasury bond ("T-bond") yields, the typical benchmark used in either the Capital Asset Pricing Model ("CAPM") or the Build-up methods of estimating COEC, are likely temporarily low, resulting in low estimates of COEC.

Second, the expected equity risk premium ("ERP"), the rate of return expected on a diversified portfolio of common stocks in excess of the rate of return on an investment in T-bonds, has likely increased as the broad stock market level has declined.

Third, because the stock market correction has been heavily concentrated in the financial services sector and in highly leveraged companies, the commonly-employed methods we use for estimating betas, the risk measure in the traditional CAPM, are potentially flawed providing faulty estimates of risk. The result is that at the very time when one assumes a priori that estimates of COEC have increased, the methods we traditionally use to estimate the COEC are providing calculations that imply risk has declined.

Fourth, current leverage ratios are likely not sustainable in the long-term for many companies and one needs to consider estimating cost of capital with expected changing capital structures.

[^0]Fifth, because income subject to income taxes is and will continue to be less than zero for many companies, one cannot automatically use an after-tax cost of debt capital (i.e., multiply the interest rate by one minus the income tax rate) in calculating an appropriate WACC.

Sixth, one must always test the resulting cost of capital estimates for reasonableness and not simply apply data or formulas by rote.

Yields on Treasury Bonds as of December 31, 2008

The general notion of a "risk-free rate" is that it is equivalent to the return available on a security that the market generally perceives as free of the risk of default as of the valuation date. ${ }^{2}$ Analysts typically use the yield to maturity on U.S. government securities as of the valuation date, as proxy for the risk-free rate.

Conceptually, the risk-free rate reflects a return on the following three components:
Rental rate: A real return for lending the funds over the investment period, thus foregoing consumption for which the funds otherwise could be used; Inflation: The expected rate of inflation over the term of the risk-free investment; Maturity risk or investment rate risk: The risk that the principal's market value will rise or fall during the period to maturity as a function of changes in the general level of interest rates. ${ }^{3}$

All three of these economic factors are embedded in the yield to maturity for any given maturity length. However, it is not possible to observe the market consensus about how much of the total yield for any given maturity is attributable to each of these factors.

Note that the risk-free rate includes inflation expectations. Therefore, when this rate is used to estimate a cost of capital to discount expected future cash flows, those future cash flows also should reflect the expected effect of inflation. In the economic sense of nominal versus real dollars, we are building a cost of capital in nominal terms, and it should be used to discount expected returns that also are expressed in nominal terms.

[^1]In valuing "going concern" businesses and long-term investments made by businesses, practitioners generally use long-term government bonds as the risk-free security and estimate the ERP in relation to long-term government bonds. This convention represents a realistic, simplifying assumption. Most business investments have long durations and suffer from a comparable reinvestment risk as long-term government bonds. As such, the use of long-term government bonds and an ERP estimated over those long-term bonds more closely matches the investment horizon and risks confronting business managers in capital decisions and valuators in valuation analyses relative to the use of Treasury bills ("T-bills"). The consensus for financial analysts today is to use the 20-year U.S. T-bonds yield to maturity as of the effective date of valuation.

Some analysts use either a 10-year or a 30-year T-bond yield; in theory one should then develop ERP estimates based on expected returns in excess of the yields for those maturities. However, as a practical matter these yields usually do not differ greatly from the 20-year yield on T-bonds. ${ }^{4}$

In applying the CAPM or the Build-up method, the analyst typically begins with the Tbond yield to maturity and adds an estimate of the ERP (in the case of the CAPM, the ERP estimate is multiplied by the risk factor beta). The ERP estimates using historical data are typically measured relative to the T-bond yield.

Since 2004, yields on 20-year (constant maturity) T-bonds have been:

| 2004 | (average for 12 months) | $5.02 \%$ |
| :--- | :--- | :--- |
| 2005 | (average for 12 months) | $4.62 \%$ |
| 2006 (average for 12 months) | $4.98 \%$ |  |
| 2007 | (average for 12 months) | $4.87 \%$ |
| 2008 | (average - first 8 months) | $4.52 \%$ |
| 2008 (September 30) | $4.43 \%$ |  |
| 2008 (October 31) | $4.78 \%$ |  |
| 2008 (November 30) | $3.72 \%$ |  |
| 2008 (December 31) | $3.03 \%$ |  |

[^2]December 31, 2008, is a particularly important date because many valuations are performed as of the end of the calendar year, thus requiring COEC to be estimated as of that date.

Most analysts would agree that the world economies are in crisis. Financial crises are often accompanied by a "flight to quality" such that the nominal returns on "risk-free" securities fall dramatically for reasons other than inflation expectations. Recent macroeconomic research suggests that inflation expectations are fairly stable, and therefore the dramatic decline in the T-bond yields in November and December 2008 is unlikely due to expected declines in expected long-term inflation. ${ }^{5}$ In fact, long term (10year horizon) Consumer Price Index (CPI) expectations continued to be at 2.5 percent at the end of $2008 .{ }^{6}$

While short-term inflation expectations have decreased, ${ }^{7}$ many commentators are warning that long-term inflation will increase, not decrease, given the projected U.S. budget deficit. Based on surveys of professional forecasters, yields on long-term U.S. government bonds are also expected to increase. For example, 10-year T-bond yields are expected to increase 1.67 percent between the spot yield on December 2008 and the end of December 2010, returning to a more "normal level" comparable to the months prior to November 2008. ${ }^{8}$

Another independent provider of economic forecasts the yield to maturity on the 30-year U.S. T-bond to increase approximately 1.5 percent from December 2008 to August 2009, as presented in the following table:

[^3]
## 30-Year U.S. Treasury Bond Yield Forecast

| Date | Forecast Value (1) |
| :---: | :---: |
| December 2008 | $2.87 \%$ |
| January 2009 | $3.00 \%$ |
| February 2009 | $3.05 \%$ |
| March 2009 | $3.15 \%$ |
| April 2009 | $3.31 \%$ |
| May 2009 | $3.50 \%$ |
| June 2009 | $3.94 \%$ |
| July 2009 | $4.19 \%$ |
| August 2009 | $4.42 \%$ |

(1) 30-year maturity secondary market rate. Percent average of month.
Source: www.forecasts.org

Further, the implied forward volatility (based on options on exchange traded funds or "ETFs") on 20-year T-bonds in November and December 2008 increased significantly (as shown in the following table), suggesting that the market is uncertain that the lower yields (and correspondingly higher prices) are sustainable.

| Ticker: <br> Description: | SPY |  | TLT |  |
| :---: | :---: | :---: | :---: | :---: |
|  | S\&P 500 ETF |  | iShares Lehman 20+ Year Treasury Bond |  |
|  | Implied $V$ | Volatility | Implied | Volatility |
| As of: | 30 Day (1)3 | 3 Mnth (2) | 30 Day (1) | 3 Mnth (2) |
| 12/31/2007 | 21.525 | 22.604 | 14.952 | 14.356 |
| 1/31/2008 | 26.121 | 23.983 | 17.578 | 16.294 |
| 2/29/2008 | 24.581 | 24.925 | 17.807 | 17.305 |
| 3/31/2008 | 25.037 | 24.59 | 16.846 | 17.239 |
| 4/30/2008 | 19.403 | 19.977 | 12.954 | 13.341 |
| 5/31/2008 | 15.929 | 18.885 | 13.081 | 14.165 |
| 6/30/2008 | 22.804 | 22.508 | 11.516 | 12.966 |
| 7/31/2008 | 22.058 | 21.838 | 11.085 | 12.316 |
| 8/31/2008 | 19.111 | 21.246 | 10.759 | 12.133 |
| 9/30/2008 | 39.166 | 31.297 | 18.686 | 16.118 |
| 10/31/2008 | 52.078 | 46.356 | 16.809 | 18.464 |
| 11/30/2008 | 51.756 | 48.393 | 28.837 | 31.087 |
| 12/31/2008 | 36.267 | 37.567 | 31.332 | 31.213 |

Notes:
(1) 30 Day Implied Volatility
(2) 3 month Implied Volatility

Source: Bloomberg

In summary, the evidence suggests that the yield on T-bonds represents an aberration as of December 31, 2008, overly influenced temporarily by the "flight to quality.

What should the analyst do when estimating the appropriate risk-free rate in developing the COEC? This author suggests that one approach is to ignore the December 31, 2008, "spot" yield on 20-year T-bonds and use a longer-term average T-bond yield (e.g., 4.5 percent) ${ }^{9}$ in developing an estimate of COEC, until such time when yields return to a more normalized level. One should then match the T-bond yield with the appropriate conditional ERP estimate for this stage in the business cycle. ${ }^{10}$

[^4]
## Equity Risk Premium

A long-term study of realized premiums in excess of the return on T-bonds indicates that realized premiums, on the average, have decreased as the T-bond yields decrease. ${ }^{11}$ But these are not ordinary times. If one simply adds an estimate of the ERP derived during "normal" economic times to the "spot" yield on 20-year T-bonds on December 31, 2008, one will likely arrive at too low of an estimate of the COEC.

As is explained in Cost of Capital $3^{\text {rd }}$ ed.,

The evidence presented above [that the long-run ERP is between 3.5\% and 6\%] represents a long-term average or unconditional estimate of the ERP. That is, what is a reasonable range of ERP that can be expected over an entire business cycle? Where in this range is the current ERP? Research has shown that ERP is cyclical during the business cycle. We use the term "conditional ERP" to mean the ERP that reflects current market conditions. For example, when the economy is near or in recession (and reflected in recent relatively low returns on stocks), the conditional ERP is more likely at the higher end of the range. When the economy improves (with expectations of improvements reflected in recent increasing stock returns), the conditional ERP moves toward the mid-point of the range. When the economy is near its peak (and reflected in recent relatively high stock returns), the conditional ERP is more likely at the lower end of the range. ${ }^{12}$

As the stock market has fallen in late 2008, the ERP implied by the S\&P 500 has increased. ${ }^{13}$ In one analysis, the implied ERP has risen to the high end of the range cited in the above quote. ${ }^{14}$

[^5]What should the analyst do in estimating the ERP? This author suggests that, given current market conditions, one should consider using an estimated ERP of 6.0 percent, the upper end of the range of the research on long-term (normal) ERP. ${ }^{15}$ As expected economic conditions improve and stock prices increase; the ERP can be expected to decrease in the future.

## Beta Estimates

The following page contains a summary of beta estimates for a sample publicly traded company ("Sample Company \#1"). Sample Company \#1's market capitalization has ranged between $\$ 1$ billion and $\$ 3$ billion in recent years. The company has zero longterm debt so its beta estimates only operating risk and no financial risk. We are displaying various beta estimates at each month end from December 2006 to December 2008.

|  |  |  |  | Beta Est | S for | ple Co | pany \#1 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| As of | $\begin{aligned} & 60 \text { Month } \\ & \hline \text { OLS Beta } \end{aligned}$ | $\frac{\mathrm{R}^{2}(60 \text { Month }}{\underline{\text { OLS Beta) })}}$ | $\begin{aligned} & \text { Total Beta } \\ & \text { (60 Month } \\ & \text { OLS Beta) } \end{aligned}$ | $\begin{aligned} & 260 \text { Week } \\ & \text { OLS Beta } \end{aligned}$ | $\frac{\mathrm{R}^{2}(260 \text { Week }}{\underline{\text { OLS Beta) }}}$ | $\begin{aligned} & \text { Total Beta } \\ & \text { (260 Week } \\ & \text { OLS Beta) } \end{aligned}$ | $\begin{array}{\|c\|} \text { Projected } \\ \text { Barra Beta } \\ \hline \end{array}$ | 60 MonthSum Beta <br> from <br> $\frac{\text { Research }}{\text { Insight }}$ | $\mathbf{R}^{2}$ (Sum Beta) | $\begin{aligned} & \text { Total Beta } \\ & \text { (Sum Beta) } \\ & \hline \end{aligned}$ |
| 12/30/2008 | 1.317 | 0.212 | N/A | 0.961 | 0.188 | 2.216 | 1.265 | 1.29 | 0.26 | 2.530 |
| 11/30/2008 | 1.299 | 0.242 | 2.641 | 0.937 | 0.183 | 2.190 | 1.014 | 1.15 | 0.23 | 2.398 |
| 10/30/2008 | 1.284 | 0.226 | 2.701 | 0.873 | 0.131 | 2.412 | 0.984 | 1.14 | 0.23 | 2.377 |
| 9/30/2008 | 1.801 | 0.283 | 3.385 | 1.143 | 0.139 | 3.066 | 1.137 | 1.75 | 0.28 | 3.307 |
| 8/30/2008 | 1.771 | 0.241 | 3.608 | 1.115 | 0.135 | 3.035 | 1.210 | 1.73 | 0.24 | 3.531 |
| 7/30/2008 | 1.740 | 0.247 | 3.501 | 1.097 | 0.131 | 3.031 | 1.091 | 1.74 | 0.25 | 3.480 |
| 6/30/2008 | 1.779 | 0.242 | 3.616 | 1.120 | 0.140 | 2.993 | 1.206 | 1.76 | 0.24 | 3.593 |
| 5/30/2008 | 2.113 | 0.274 | 4.037 | 1.124 | 0.137 | 3.037 | 1.259 | 2.12 | 0.27 | 4.080 |
| 4/30/2008 | 2.323 | 0.312 | 4.159 | 1.163 | 0.141 | 3.097 | 1.360 | 2.50 | 0.31 | 4.490 |
| 3/30/2008 | 2.479 | 0.371 | 4.070 | 1.252 | 0.160 | 3.130 | 1.301 | 2.60 | 0.37 | 4.274 |
| 2/29/2008 | 2.492 | 0.349 | 4.218 | 1.095 | 0.131 | 3.025 | 1.341 | 2.77 | 0.35 | 4.682 |
| 1/30/2008 | 2.482 | 0.340 | 4.257 | 1.186 | 0.142 | 3.147 | 1.298 | 2.78 | 0.35 | 4.699 |
| 12/30/2007 | 2.348 | 0.272 | 4.502 | 1.105 | 0.122 | 3.164 | 1.336 | 2.28 | 0.27 | 4.388 |
| 11/30/2007 | 2.460 | 0.326 | 4.309 | 1.132 | 0.128 | 3.164 | 1.330 | 2.54 | 0.32 | 4.490 |
| 10/30/2007 | 2.450 | 0.322 | 4.318 | 1.174 | 0.130 | 3.256 | 1.384 | 2.54 | 0.32 | 4.490 |
| 9/30/2007 | 2.842 | 0.399 | 4.499 | 1.433 | 0.201 | 3.196 | 1.540 | 2.44 | 0.41 | 3.811 |
| 8/30/2007 | 2.930 | 0.500 | 4.144 | 1.526 | 0.232 | 3.168 | 1.334 | 2.56 | 0.51 | 3.585 |
| 7/30/2007 | 2.944 | 0.492 | 4.197 | 1.496 | 0.223 | 3.168 | 1.404 | 2.79 | 0.49 | 3.986 |
| 6/30/2007 | 2.707 | 0.460 | 3.991 | 1.414 | 0.208 | 3.100 | 1.408 | 2.40 | 0.47 | 3.501 |
| 5/30/2007 | 2.945 | 0.515 | 4.104 | 1.516 | 0.214 | 3.277 | 1.422 | 2.62 | 0.52 | 3.633 |
| 4/30/2007 | 3.007 | 0.533 | 4.119 | 1.512 | 0.218 | 3.238 | 1.326 | 2.80 | 0.53 | 3.846 |
| 3/30/2007 | 3.057 | 0.559 | 4.089 | 1.577 | 0.232 | 3.274 | 1.489 | 2.86 | 0.56 | 3.822 |
| 2/28/2007 | 3.011 | 0.550 | 4.060 | 1.578 | 0.227 | 3.312 | 1.392 | 2.85 | 0.55 | 3.843 |
| 1/30/2007 | 3.099 | 0.540 | 4.217 | 1.592 | 0.215 | 3.433 | 1.512 | 3.00 | 0.54 | 4.082 |
| 12/30/2006 | 3.072 | 0.535 | 4.200 | 1.598 | 0.214 | 3.454 | 1.465 | 2.97 | 0.53 | 4.080 |

[^6]Prior to May 2008, the ordinary least squares regression estimate of beta ("OLS Beta") was 2.0 or greater. ${ }^{16}$ These estimates result from a regression and are made "with estimation error." Total Beta has exceeded 4.0 during that same period ${ }^{17}$. One interpretation of Total Beta is a beta estimate corrected for estimation error.

But after May 2008 we see OLS Beta estimates have decreased to a range of approximately 1.3 to 1.8 .

What happened? Overall stock market indices such as the S\&P 500 have been overly influenced by financial stocks and stocks of highly leveraged companies. The relative volatility of returns for Subject Company \#1 with no debt has declined relative to a market whose returns (negative) are over-weighted by financial companies. The Subject Company \#1 business risk relative to the overall economy did not change during this period. But relative to a market over-weighted by financial companies, it appears to have decreased in risk.

The graph below helps explain these relationships. One can see the severe downward adjustment to the financial sector stocks, which initially dragged the S\&P 500 down even as the other sectors were bouncing back. Ultimately, other sectors followed suit as economic conditions in other sectors of the economy deteriorated further.

[^7]

During these past months, we have in essence observed a process of re-pricing of the stock market in general and, in particular, of many stocks at new lower prices. The low beta estimates for some stocks, such as Subject Company \#1, derived from analyzing stock returns during a "look-back period" result from the negative returns on the stock market portfolio and many other stocks as the stock market seeks its new, lower equilibrium price. The low beta estimate currently observed above is not from a change in the underlying long-term relative business risk of Subject Company \#1 to the business risk of the economy as represented by the stock market. For example, prices of financial sector stocks (and their returns) have trended downwards looking for new equilibrium levels; once those levels are reached, the relative volatility of these stocks to the stock market will return to "normal". But during this adjustment period, prices of stocks such as Sample Company \#1 have moved downward relatively little (or not as much as the market portfolio), making their observed beta estimates lower than historic norms and lower than what one might expect in the future after the market portfolio is finished repricing at a new, lower equilibrium level.

While such adjustments in pricing occur for some stocks during all time periods, over these past few months we have seen the stock market (as represented by the S\&P 500 for example) experience a major re-pricing led by financial sector stocks and highly
leveraged non-financial stocks. Stocks of companies with traditionally high operating leverage (operating income and prices moving up faster than the overall market during upward market price movements, and moving down faster than the market when the market declines) appear to indicate that operating leverage has decreased when in fact their underlying operating leverage has not changed.

The best way to identify and observe the condition just described is to graph the returns of a particular company (or industry) over time relative to the overall market. The following graph presents an example of an adjustment in pricing for a hypothetical sample company:

Example Company Vs. Index
Over Time


In period $A$, the sample company essentially moves with the market. In period $B$, the sample company is experiencing a downward re-pricing, and during this period the sample company's returns are not as strongly correlated with the movement of the overall market. In Period C, the re-pricing of the sample company is complete, and the sample company's returns are once again moving in tandem with market returns.

If one were to compute beta at Time 1 , which includes period " $A$ " as the "look-back" period, the beta estimate would reflect the normal relationship between the sample company's returns in the market's returns. In contrast, computing a beta estimate at

Time 2, which includes period "B" (the sample company's re-pricing by the market) as the "look-back" period, would not yield a reliable forward-looking beta estimate. In fact, it would yield a beta estimate lower than expected since the sample company's return was negative in a period when the market was generally rising. This result is counter-intuitive given the sample company's downward re-pricing, i.e., the operating risk of the sample company has not declined over period "B" and will resume its "normal" relationship to the market in period "C."

Returning to Sample Company \#1, the lower beta estimate reflects that stock's lower risk during the market's adjustment period (particularly since Sample Company \#1 has no debt). But looking forward to periods following the market's re-pricing, one must assess whether the true beta of Sample Company \#1 (i.e., the expected relationship of returns for Subject Company \#1 to changes in the economy as represented by a stock market index like the $S \& P 500$ ) will be better represented by the longer term beta estimate or the recent lower estimate measured from a date like Time 2 over a recent look-back period.

One should consider examining alternative beta estimation methods, such as Sum Beta estimates. Sum Beta estimates generally result in more accurate (higher) estimates of beta for smaller capitalization companies, ${ }^{18}$ and in the current environment, as market capitalizations have decreased, more companies are considered small and midcapitalization companies.

For those who subscribe to Barra ${ }^{19}$ projected beta information services, one can verify whether Barra's projected beta estimates correct for this phenomenon. We observe that Barra projected beta estimates for Sample Company \#1 were 1.3 or greater before May 2008, while the most recent Barra projected beta estimates are just approximately 1.0 reverting back to 1.3 only in December 2008.

Barra projected estimates for Sample Company \#1 suffer from two problems. First, Barra projected beta estimates begin with OLS Beta estimates. If one is getting faulty beta

[^8]estimates from OLS Beta estimates, those faulty signals will likely carry over to the Barra projected beta estimates. Second, Barra projected beta estimates appear to not correctly adjust for errors in estimating the betas of smaller cap companies. ${ }^{20}$ Barra projected beta estimates seem to accentuate the problem of underestimation with beta estimates for small capitalization companies. This will become a more pronounced problem as the market value of equity has declined substantially for most companies in the current market downturn.

Do the "adjusted beta" estimates provided by Bloomberg provide an alternative? Those estimates are not really adjusted the way one thinks of "adjusted" - changed based on specific characteristics of the company. Rather, Bloomberg adjusted beta estimates are somewhat arbitrarily adjusted toward 1.0, under the premise that eventually every company's beta will converge to the market beta; this adjustment is not therefore based on specific industry or company factors.

The table below contains a summary of beta estimates for a sample publicly traded company ("Sample Company \#2"). Sample Company \#2 is not in the financial sector. Its market capitalization has ranged between $\$ 10$ billion and $\$ 29$ billion in recent years, and the company has considerable long-term debt so its beta estimates reflect both financial and operating or business risk. We are displaying various beta estimates at each month end from December 2006 to December 2008.

[^9]

In this example, we observe that the beta estimates in late 2008 generally mirror the beta estimates of late 2007 and early 2008. But during the initial phases of the current economic and credit crisis, Sample Company \#2's beta estimate decreased because this company is not in the financial sector. Later, as the financial crisis spread to leveraged companies in all industries, the beta estimates increased accordingly.

What should the analyst do to estimate an appropriate beta? This author suggests that one start by graphing the monthly returns for the subject company and the S\&P 500 (both measured on the " $y$ " axis) over time (measured on the " $x$ " axis) for the last 24-36 months. ${ }^{21}$ One can then verify if and when the underlying relationship between returns for the subject company and returns for the market may have changed.

One might then consider taking the average of the month-end beta estimates over, say, a 12-month period during which the relationship appears to be more "normal" (in the case of the Sample Company \#1, that period may be the period from April 2007 through May 2008). This is the beta estimate that one might reasonably expect going forward, once the stock market has completed its re-pricing to a new, lower equilibrium price.

[^10]Regardless of the methodology or the data service used for beta estimates, one must remember that beta is an estimate of the expected future relationship between changes in the returns on the subject company's stock to changes in the stock market returns. In other words, the application of CAPM requires the use of a forward-looking beta as a measure of future risk. As such, one must be cautious that the estimates make sense relative to the underlying risk of the stock and not simply rely on "spot" estimates using a single beta estimation methodology derived from returns during a "look-back" period that may not represent the expected relationship of returns in future periods. ${ }^{22}$

## Leverage - Impact on Beta Estimates

Beta estimates derived from the relationship of observed stock returns to market returns are a function of all risks affecting a company: both operating leverage (change in operating earnings as the market for the company's products increases and decreases) and financial leverage (the added variability in net income and stock returns because the company finances its investments partially with long-term debt capital). If one is estimating the COEC for a public company, one can use the observed relationship of returns on that company's stock relative to returns on the market portfolio over a "lookback" period to help make a forward beta estimate, based on the company's current amount of debt financing. But if one is estimating the COEC assuming that the current level of debt will actually change, then the first step should be to "un-lever" the beta estimate (removing the effect of financial risk from the beta estimates) for the subject company, to arrive at what is often called an "asset beta" estimate for the subject public company.

If one is estimating the COEC for a reporting unit of a public company (e.g., for goodwill impairment testing under Statement of Financial Accounting Standard No. 142) or for a closely-held company, one must use beta estimates from guideline public companies as a proxy beta estimate for the subject reporting unit or closely-held company. ${ }^{23}$ One first

[^11]"un-levers" the proxy beta estimates for the guideline public companies to arrive at an "asset beta" estimate.

An underlying principle that one must remember is that we are looking to measure the risk of the subject public company, subject reporting unit or closely-held company and determine the appropriate cost of capital for the associated risk.

In the case of a public company, one "re-levers" the asset beta to reflect the financing structure a potential acquirer may use or a target debt structure for the subject company.

In the case of reporting units of a public company, one "re-levers" the un-levered beta estimate for the appropriate leverage that market participants (companies in the pool of possible acquirers for the reporting unit) would use in valuing the reporting unit. In determining the appropriate leverage, one must consider: (1) which companies comprise the pool of likely market participant buyers (because the premise to be taken into account in testing for goodwill is a hypothetical "exit price" premise, i.e. what is the appropriate cost of capital as if the reporting unit were sold as of the "testing date"); and (2) how would those market participant buyers finance the purchase of the reporting unit. One cannot assume that if the market participant buyers have a lower cost of capital they would price the acquisition of a reporting unit using their own lower cost of capital; doing so is equivalent to transferring value to the hypothetical seller. If the reporting unit is economically distressed (i.e., operating income is suffering) or the company owning the reporting unit is financially distressed (i.e., there is a high risk that the company may default on its debt), market participants will estimate a cost of capital in valuing the reporting unit which appropriately reflects that distress, rather than the lower cost of capital of the market participant's own business.

In the case of a closely-held company, one does not know the market value of the closely-held company until the valuation process is completed, but the re-levered COEC is dependent upon the ratio of debt to equity capital measured at market value, one must apply an iterative process to determine the appropriate re-levered beta and COEC. ${ }^{24}$

[^12]Analysts typically use standard formulas for un-levering observed beta estimates. Such un-levering in theory removes the effect of financial leverage, and all that remains is the expected variability in stock returns due to operating leverage.

Once analysts conclude on a reasonable asset beta estimate for the subject business, then the analyst may re-lever the beta to an appropriate debt level based on the debt capacity of the subject business. The debt capacity may be represented by an industry average debt level, for example, if the analyst were estimating the value of a reporting unit in terms of market participants, or a target debt level, for example, if the analyst were estimating the value of the subject company. But one should not automatically assume that historical debt levels represent current debt capacity. Rather, one needs to analyze the expected available cash flows given the likely lower expectations in the current economic environment.

The typical "textbook" un-levering and re-levering formulas used are based on more stable times. For example, the "Hamada formula," which is often (mis-) used, will be particularly problematic as this model assumes (1) that the current debt remains constant over time; and (2) the company will realize all income tax deductions on interest expense in the period in which the interest on debt is paid. ${ }^{25}$ Implicit in this formula is the assumption is that debt beta is zero and tax shields are certain. During the current period of economic crisis, we have seen the percentage of debt to equity (at market values) rise dramatically, as equity values have shrunk thereby increasing the risk of realizing tax deductions in the period in which interest is paid. Consequently, analysts should consider other models of unlevering.

The "Miles-Ezzell formula" is an appropriate formula for un-levering and re-levering beta estimates when the underlying assumption holds that a constant debt-to-equity (at market value weights) capital structure will be maintained. That formula does adjust for the impact of joint risk taking between debt capital and equity capital through the introduction of (1) a beta on debt greater than zero; and (2) the risk that tax benefits from interest deductions will not be realized in the period in which the interest is paid. ${ }^{26}$ The underlying assumptions are that debt beta is positive and tax shields are certain for only

[^13]one period and uncertain afterwards. The Miles-Ezzell formula for un-levering beta is as follows:
$$
B_{U}=\frac{M_{e} \times B_{L}+M_{d} \times B_{d}\left\lfloor 1-\left(t \times k_{d(p t)}\right) /\left(1+k_{d(p t)}\right)\right]}{M_{e}+M_{d}\left[1-\left(t \times k_{d(p t)}\right) /\left(1+k_{d(p t)}\right)\right]}
$$
where:

| $B_{U}$ | $=$ Unlevered beta of equity capital |
| :--- | :--- |
| $B_{L}$ | $=$ Levered beta of equity capital |
| $M_{e}$ | $=$ Market value of equity capital (stock) |
| $M_{d}$ | $=$ Market value of debt capital |
| $B_{d}$ | $=$ Beta of debt capital |
| $t$ | $=$ Income tax rate for the company |
| $k_{d(p t)}$ | $=$ Cost of debt prior to tax affect |

The companion Miles-Ezzel formula for re-levering beta is as follows:

$$
B_{L}=B_{U}+\frac{W_{d}}{W_{e}}\left(B_{U}-B_{d}\right)\left[1-\frac{\left(t \times k_{d(p t)}\right)}{\left(1+k_{d(p t)}\right)}\right]
$$

Debt betas can be measured using an estimation method over a "look-back" period.
One can estimate the beta on debt based on a particular credit rating (either actual credit rating or a synthetic credit rating ${ }^{27}$ ). For example, the estimated debt betas by credit rating for U.S. corporate and high-yield long-term bond series as of December 31, 2008 are as follows: ${ }^{28}$

[^14]| Aaa | 0.123 |
| :--- | :--- |
| Aa | 0.177 |
| A | 0.355 |
| Baa | 0.428 |
| Ba | 0.685 |
| B | 0.774 |
| Caa | 1.105 |
| Ca-D | 1.511 |

Debt beta estimates change over time and these current debt betas have increased relative to debt beta estimates in earlier years (as the current market considers debt capital financing to be more risky today). This makes the use of the correct un-levering formula more critical.

But even the Miles-Ezzell formula may understate the risk brought on by debt levels relative to the market value of equity. There are alternate formulas one should consider. ${ }^{29}$ Debt levels have increased (as equity has been re-priced downward), decreasing the likelihood that the tax benefits of debt financing will be fully realized.

The affect of increasing debt levels is that the COEC likely is understated by using any of the traditional un-levering formulas. All of the formulas define linear relationships. Research indicates that the correct relationship is not linear as leverage increases; rather the COEC increases at an increasing (or exponential) rate as leverage increases.

The following graph displays the likely market relationship of debt and equity betas as the level of debt increases. ${ }^{30}$ In this market, leverage is increasing just because stock market capitalizations are decreasing.

[^15]

As the levels of debt to equity (measured at market values) increase, the costs of financial distress increase as well (value lost due to the increase in the chance of default induced by the firm's debt adjusted for the present value of the expected tax deductions on interest payments on the debt). One study quantifies the cost of economic distress at varying levels of debt. ${ }^{31}$

The Duff \& Phelps Risk Premium Report provides data on realized equity returns in excess of the returns predicted by CAPM for "High Financial Risk" companies. ${ }^{32}$ This premium can be added to the standard CAPM estimate of the increase in the COEC for the market's estimate of the cost of distress (economic and financial distress). The premiums over CAPM as of December 31, 2007, were approximately 6.3 percent to 6.5 percent.

[^16]What should the analyst do relative to adjusting beta estimates for leverage? For companies using debt financing, one should estimate (i) the market value of the debt, (ii) the debt rating on the debt (either actual or synthetic based on coverage ratios published by ratings companies such as Standard \& Poor's or Moody's) and (ii) the appropriate unlevered or asset beta using the Miles-Ezzell formula.

Once analysts conclude on a reasonable asset beta estimate for the subject business, then the analyst may re-lever the beta with the same formula to an average debt level market participants would use (for example, if the analyst were estimating the value of a reporting unit) or a target debt level (for example, if the analyst were estimating the value of the subject company, knowing that the current level of debt must be reduced over the long-term).

If the subject company at the assumed debt level is in distress, then one needs to consider adjusting the indicated COEC arrived at using standard techniques to adjust for the costs of distress.

But assume that we are valuing a subject company that is in such financial distress that the value of the assets (measured as the present value of expected net cash flows using the unlevered cost of equity capital) appears to be less than the face value of debt. Would anyone be willing to pay anything to acquire the equity? In essence, will the future value of equity possibly exceed the face value of debt? By estimating (1) the value of the possibility that the value of the business without regard to the current amount of debt will exceed the face value of debt at some future point in time and (2) the probability that this will occur at some future point in time, one is explicitly considering the right "tail" of the probability distribution of future net cash flows. The valuation of the subject company can be cast as a scenario analysis of discounted cash flows with the probability of each scenario or an option analysis. ${ }^{33}$

[^17]
## WACC and the Value of the Tax Shield

The textbook formula for developing the WACC is as follows:

$$
W A C C=\left(k_{e} \times W_{e}\right)+\left(k_{p} \times W_{p}\right)+\left(k_{d(p t)}[1-t] \times W_{d}\right)
$$

where:

WACC = Weighted average cost of capital (after-tax)
$k_{e} \quad=$ Cost of common equity capital
$W_{e} \quad=$ Percentage of common equity in the capital structure, at market value
$k_{p} \quad=$ Cost of preferred equity
$W_{p} \quad=$ Percentage of preferred equity in the capital structure, at market value
$k_{d(p t)} \quad=$ Cost of debt (pre-tax)
$t \quad=$ Income tax rate
$W_{d} \quad=$ Percentage of debt in the capital structure, at market value

This textbook formula assumes that (1) tax deductions will be realized on interest payments in the period in which they are accrued, (2) earnings before interest and taxes (plus other income) are greater than financial expenses and the full tax shield will be earned. (3) market value of debt is equal to its book value and, hence, the contractual cost of debt is identical to the market cost of debt.

The correct analysis does not automatically multiply the interest rate by one minus the income tax rate. Graphically, we can display the correct relationship as follows.

## Value of a Levered Firm



In this formulation, cost of debt capital is measured after the tax affect $\left(k_{d}\right)$. The tax shield is the present value of the expected tax deductions, which today are likely to be more risky than in prior periods.

Do companies realize deductions at the statutory tax rate (get full benefit of interest tax deduction in the period in which the interest is paid)? Researchers have developed a simulated expected tax rate model that simulates taxable income into the future. This process has shown that many companies do not expect to pay the highest marginal rate for long periods of time. Because of tax loss carry-backs and carry-forwards and the cyclical nature of some industries, a substantial number of companies can expect a very low tax rate. ${ }^{34}$

Graham and Mills completed a simulation study of corporate marginal income tax rates. They used U.S. tax return data for public corporations from 1990 to 2000 to simulate the corporate marginal tax rates for 1998 to 2000. They used this data because financial statement data can vary greatly from tax return data. Actual taxes paid are the correct measure for the cost of debt capital, rather than taxes reported under "book" financials for accounting purposes. These authors found that the simulated marginal tax rate most

[^18]closely approximated future actual taxes paid. But when the simulated model is not available, they offer two formulas based on actual corporate income tax data to estimate the corporate marginal tax rate. ${ }^{35}$ These formulas can be useful in estimating the expected cash tax rate instead of arbitrarily using the marginal income tax rate.

As the market value of equity has declined for many companies the percentages of debt capital to equity capital have become out of equilibrium. Either the subject company will need to pay down debt (as they may or may not be able to refinance existing debt levels given actual and expected reductions in operating income many companies are experiencing) or raise equity capital to return to a long-term equilibrium where the cost of debt is manageable given operating income and the equity value is not penalized for carrying too much debt. The WACC can be applied under an assumption of changing capital structure; for example, as the debt changes over time to a target debt level, the WACC changes. In this formulation, as the debt level changes over time, the re-levered equity beta and the resulting COEC changes. ${ }^{36}$

What should the analyst do in estimating the WACC for the subject company? One must estimate the expected income tax deductions that will be realized from the payment of the interest on the level of debt capital assumed in the re-levered capital structure. During these troubled economic times, one cannot simply assume that the full tax benefit will be realized as taxable income before interest will likely be zero or negative for many companies for 2008 and 2009. The assumptions embodied in the textbook WACC formula lead one to the conclusion that companies should abandon its use.

A generalized formula for the WACC that takes into account the probability that income tax savings on interest payments will not be realized in the period in which the interest is paid is as follows: ${ }^{37}$

[^19]$$
W A C C_{t}=k_{\text {eut }}-\left\{T S_{t} /\left[M_{d t-1}+M_{e t-1}\right]\right\}-\left\{\left(k_{\text {eut }}-k_{T S}\right)\left(P V_{T S t-1} /\left[M_{d t-1}+M_{e t-1}\right]\right\}\right.
$$
where:

| $k_{\text {eut }}$ $=$ COEC, un-levered (COEC assuming firm <br>  financed with all equity) at time $=t$ |  |
| :--- | :--- |
| $T S_{t}$ | $=$ Tax shield realized at time $=t$ |
| $M_{d t-1}$ | $=$ Market value of debt capital at time $=t-1$ |
| $M_{e t-1}$ | $=$ Market value of equity capital at time $=t-1$ |
| $k_{T S}$ | $=$ Discount rate on tax shield based on the risk of realizing the tax |
|  | shield (typically either $k_{d(p t)}$, the pre-tax cost of debt, or $\left.k_{e u}\right)$ |
| $P V_{T S t-1}$ | $=$ Present value of the tax shield as of time $=t-1$ |

If we assume that $k_{T S}=k_{\text {eut }}$ (the variability of one realizing the tax shield is approximately equal to the variability of cash flows of the business before interest expense) then the above formula simplifies to:

$$
W A C C_{t}=k_{\text {eut }}-\left\{T S_{t} /\left[M_{d t-1}+M_{e t-1}\right]\right\}
$$

## Cross Checking Cost of Capital Estimates

Today's environment is making cost of capital estimation particularly challenging. How can one check for the reasonableness of their cost of capital estimates?

One check you can make on COEC estimates is to fall back on the classic Graham and Dodd. ${ }^{38}$ Their methodology was based on the yield of the bonds of the corporation (reflecting the leverage and the company-specific risks imbedded in the credit ratings) plus an average equity premium of, say, 4 percent. More recent research indicates that

[^20]this spread goes up as the debt rating decreases (the average equity spread over corporate bond yield may be 4 percent, but it is greater for low rated bonds, say 7 percent for companies whose debt is rated $B$ ).

The COEC should logically exceed the yield investors are expecting on the company's debt capital (without reducing the yield by any income tax deductions that might be realized by the subject company). Equity capital is more risky than debt capital and the market will price each component based on their relative risk. In "normal times," one would examine the spreads over T-bonds. In this environment with the yields on T-bonds artificially low, spreads are not meaningful. Rather, one should look at the absolute level of market yield on the company's debt (market yield for the debt rating on the subject company's debt level, either actual or target, based on the actual or synthetic debt rating of the subject company) and the COEC should exceed that yield on debt. ${ }^{39}$

Another course of action is to use the data provided in the Duff \& Phelps' Risk Premium Report to estimate the COEC. The Duff \& Phelps Report provides equity risk premium data for use in a build-up model that is independent of estimates of beta. In a recent survey conducted by Business Valuation Resources, over 50\% of valuation practices in the U.S. are now using the Duff \& Phelps Risk Premium Report to help them develop estimates of COEC.

Two of the exhibits in the Risk Premium Report are particularly helpful in quantifying the increase in the COEC that may be appropriate given the increased risk of operations. One exhibit displays data on historic equity returns based on companies' average operating margins; another exhibit displays data on historic equity returns based on the variability of companies' operating margins. ${ }^{40}$ On average, the lower the operating margin, the higher the business risk; and on the average, the greater the variability in operating margin, the higher the business risk. The research contained therein demonstrates that stock market participants price increased risk. In this time of uncertainty, the subject company may not just be experiencing lower levels of earnings, but also increasing variability of earnings. If the subject company is expecting lower operating margins and increasing variability in operating margins, then the COEC has

[^21]likely increased and the Duff \& Phelps Risk Premium Report provides data to help quantify the appropriate increase.

Has the COEC for most companies increased? This author believes that the market is highly divided between companies with no or limited debt and companies with high levels of debt. If one looks at the absolute yields on Aaa and Aa rated companies, one can conclude that there is likely only a small increase in the COEC and the WACC for companies with no debt or highly rated debt.

Yields

|  | 20 Year Gov TBond | IT Gov TBond | 30 Day T- Bill <br> Bill | Long-Term A Rated Corporate | Long-Term Aa Rated Corporate | Long-Term Aaa Rated Corporate | Long-Term <br> Baa Rated <br> Corporate | Long-Term High Yield Corporate |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Jan-07 | 5.02 | 3.01 | 0.44 | 6.14 | 5.90 | 5.78 | 6.47 | 7.61 |
| Feb-07 | 4.77 | 4.48 | 0.38 | 5.90 | 5.72 | 5.57 | 6.23 | 7.45 |
| Mar-07 | 4.93 | 4.51 | 0.43 | 6.11 | 5.96 | 5.73 | 6.44 | 7.59 |
| Apr-07 | 4.89 | 4.49 | 0.44 | 6.04 | 5.92 | 5.67 | 6.35 | 7.47 |
| May-07 | 5.10 | 4.83 | 0.41 | 6.24 | 6.09 | 5.84 | 6.54 | 7.48 |
| Jun-07 | 5.21 | 4.90 | 0.40 | 6.40 | 6.26 | 6.01 | 6.74 | 8.10 |
| Jul-07 | 5.01 | 4.57 | 0.40 | 6.43 | 6.35 | 5.96 | 6.83 | 9.08 |
| Aug-07 | 4.87 | 4.20 | 0.42 | 6.42 | 6.34 | 5.91 | 6.74 | 9.08 |
| Sep-07 | 4.89 | 4.13 | 0.32 | 6.37 | 6.29 | 5.88 | 6.70 | 8.64 |
| Oct-07 | 4.80 | 4.35 | 0.32 | 6.25 | 6.28 | 5.75 | 6.55 | 8.72 |
| Nov-07 | 4.45 | 3.33 | 0.34 | 6.30 | 6.33 | 5.49 | 6.61 | 9.48 |
| Dec-07 | 4.50 | 3.28 | 0.27 | 6.39 | 6.26 | 5.70 | 6.64 | 9.64 |
| Jan-08 | 4.36 | 3.01 | 0.21 | 6.45 | 6.23 | 5.86 | 6.80 | 10.04 |
| Feb-08 | 4.38 | 2.56 | 0.13 | 6.53 | 6.36 | 5.97 | 6.96 | 10.58 |
| Mar-08 | 4.32 | 2.45 | 0.17 | 6.73 | 6.50 | 6.08 | 7.11 | 10.86 |
| Apr-08 | 4.58 | 3.16 | 0.18 | 6.59 | 6.38 | 6.05 | 6.96 | 10.02 |
| May-08 | 4.75 | 3.40 | 0.18 | 6.85 | 6.61 | 6.43 | 7.08 | 10.06 |
| Jun-08 | 4.60 | 3.30 | 0.17 | 6.97 | 6.67 | 6.36 | 7.18 | 10.89 |
| Jul-08 | 4.65 | 3.22 | 0.15 | 7.20 | 6.71 | 6.51 | 7.38 | 11.43 |
| Aug-08 | 4.49 | 3.03 | 0.13 | 7.15 | 6.58 | 6.45 | 7.35 | 11.56 |
| Sep-08 | 4.43 | 2.89 | 0.15 | 8.06 | 7.26 | 8.04 | 8.19 | 13.91 |
| Oct-08 | 4.78 | 2.60 | 0.08 | 9.04 | 7.75 | 8.18 | 10.16 | 18.67 |
| Nov-08 | 3.72 | 1.61 | 0.03 | 8.34 | 6.93 | 7.16 | 9.54 | 21.83 |
| Dec-08 | 3.03 | 1.26 | 0.09 | 6.95 | 5.61 | 5.86 | 8.62 | 19.50 |

But as you look at the absolute levels of yields of lower rated corporate bonds, the absolute level of yields has increased, indicating that the COEC has increased as well.

If one examines the required returns estimated in Quantitative Profiles, one sees that the average COEC has increased. ${ }^{41}$ For example, their estimated COEC (called "required return") for firms for which they assign a quality rank of B+ or better increased from

[^22]$10.3 \%$ at the beginning of 2009 to $11.1 \%$ at the beginning of 2009. For firms for which they assign a quality rank of B or worse the estimated COEC increased from 11.9\% at the beginning 2008 to $13.6 \%$ at the beginning of 2009. These COEC estimates are measured using market risk-free rates; had the risk-free rates at the end of 2008 not been abnormally low, the increases would have been larger.

## Conclusion

Estimating the appropriate cost of capital is always difficult as pricing risk is a difficult exercise. But in today's environment it is even more challenging and requires extreme care on the part of the analyst. This author is not suggesting changing or straying from the traditional models typically employed in estimating the COEC, but rather is advising analysts to take a closer look at the inputs that go into these models. Likely temporary aberrations in several of the inputs to traditional models during this period of economic crisis require analysts to apply more rigor and scrutiny in developing cost of capital estimate. ${ }^{42}$ Any concluded cost of capital estimation must balance the correct application of various models, including associated inputs, with the basic theory of finance and common sense over the long-term.

I hope this article helps you identify problems with applying the standard techniques used in estimating cost of capital and provide some ideas on improving and testing your cost of capital estimates.

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[^23]Roger Grabowski is a Managing Director in the Chicago office of Duff \& Phelps, LLC. This paper does not represent the official position of Duff \& Phelps, LLC.


[^0]:    ${ }^{1}$ This is an edited version (reflecting edits through Feb 25, 2009) of the article that appeared in the Business Valuation E-Letter, Issue 13-05 and an update to an article that appeared in the Business Valuation E-Letter, Issue 12-44 published by the American Society of Appraisers.

[^1]:    ${ }^{2}$ Shannon Pratt and Roger Grabowski, Cost of Capital: Applications and Examples $3^{\text {rd }}$ ed. (Wiley, 2008), Chapter 7.
    ${ }^{3}$ This risk gives rise to the so-called horizon premium.

[^2]:    ${ }^{4}$ It is also noted that the 30-year T-bond was characterized in several periods during the 1990's and 2000's by a lower yield-to-maturity than the 10-year T-bond. This was partially attributable to a lack of 30-year bond issuance by the US government, which resulted in a downward kink in the yield curve - this was not necessarily reflective of long-term risk perceptions, but rather a function of supply and demand on the 30year T-bonds.

[^3]:    ${ }^{5}$ V.V. Chari, Lawrence Christiano and Patrick J. Kehoe, "Facts and Myths about the Financial Crisis of 2008," Federal Reserve Bank of Minneapolis Research Department working paper 666 (October 2008)
    6 "Survey of Professional Forecasters", Federal Reserve Bank of Philadelphia, November 17, 2008; "The Livingston Survey," Federal Reserve Bank of Philadelphia, December 9, 2008.
    7 "The Livingston Survey," Federal Reserve Bank of Philadelphia Research Department comparing the projected increases in the producer price index for 2009 contained in June 2008 and December 2008 surveys.
    ${ }^{8}$ Ibid, December 9, 2008.

[^4]:    ${ }^{9}$ Alternatively, one could use a "forward" rate on T-bonds.
    ${ }^{10}$ If one uses the apparently abnormal spot yield on 20-year T-bonds as of December 31, 2008, in developing one's estimate of the COEC then one should use an ERP estimate consistent with the abnormal spot yield; see footnote 13 and Aswath Damodaran, "What is the riskfree rate? A Search for the Basic Building Block," working paper (December 2008).

[^5]:    ${ }^{11}$ Aswath Damodaran, "Equity Risk Premiums: Determinants, Estimation and Implications," (September 2008 with an October update reflecting the market crisis), pp. 56-57.
    ${ }_{12}$ Pratt and Grabowski, op. cit., Chapter 9.
    ${ }^{13}$ Damodaran, op. cit., pp. 54. The implied ERP is the discount rate that equates the S\&P 500 index with expected dividends plus stock buybacks.
    ${ }_{14}$ Damodaran On-Line Update, January 2009. Damodaran reports that the implied ERP as of January 1, 2009, equals 6.43 percent (measured from the "below normal" yield on 10 -year T-bonds) while the ERP estimate based on historic returns equals 3.88 percent. The implied ERP at January 26 stood at approximately 7 percent (measured from the "below normal" yields on 10-year T-bonds).

[^6]:    ${ }^{15}$ If one uses the apparently abnormal spot yield on 20-year T-bonds as of December 31, 2008, in developing one's estimate of the COEC and a higher ERP estimate consistent with the abnormal spot yield, one will need to update (reduce) their ERP estimate once spot yields return to more normal levels and not simply adjust their ERP estimate annually as is common practice.

[^7]:    ${ }^{16}$ Estimating beta over a 60-month "look-back" period; see Pratt and Grabowski, op.cit., Chapter 10.
    ${ }^{17}$ Total Beta equals [beta / R] or [ $\sigma_{s} / \sigma_{m}$ ], the relative standard deviation of the returns on the subject security to the standard deviation of returns on the market. See: Chris Tofallis, "Investment Volatility: A Critique of Standard Beta Estimation and a Simple Way Forward," working paper (January 2008).

[^8]:    ${ }^{18}$ Pratt and Grabowski, op. cit., Chapter 10 and Appendix 10-B. The formula on page 154 contains a typographical error and should read: Market Lagged Coefficient $=+[\operatorname{Varp}($ Market $)$ * Covar(Company,Lagged) - Covar(Market, Lagged) * Covar(Company,Lagged) ]/ (Varp(Market) * Varp(Lagged) - Covar(Market,Lagged)^2)
    ${ }^{19}$ Barra is now part of MSCI Barra. MSCl acquired Barra International in 2004.

[^9]:    ${ }^{20}$ Pratt and Grabowski, op. cit., pp 136-138.

[^10]:    ${ }^{21}$ This is not the typical graph of the returns with the S\&P 500 on the " $x$ " axis and the returns of the subject stock on the " $y$ " axis. Rather, what is being suggested is a graph over time.

[^11]:    ${ }^{22}$ Beta estimation techniques continue to be the subject of research. For example, one working paper suggests that beta estimates based on short look-back periods are negatively correlated to future returns while beta estimates based on longer look-back periods are better correlated to future returns. See, Gerard ${ }_{23}$ Hoberg and Ivo Welch, "Long-Tem and Short-Term Market Betas in Securities Prices" (May 17, 2007). ${ }^{23}$ Unless the risk of the reporting unit closely resembles that of the publicly traded company to which it belongs. In such a case, the asset beta of the subject company is the best proxy for the reporting unit's asset beta.

[^12]:    ${ }^{24}$ Pratt and Grabowski, op. cit., Appendix 17A explains the iterative process for a constant capital structure and Appendix 17B for a changing capital structure.

[^13]:    ${ }^{25}$ Pratt and Grabowski, op. cit., pp 143-145.
    ${ }^{26}$ Pratt and Grabowski, op. cit., pp 144-147.

[^14]:    ${ }^{27}$ A synthetic debt rating is developed by the analyst from comparing coverage ratios for debt instruments rated by a rating service such as Moody's or Standard \& Poor's.
    ${ }^{28}$ Pratt and Grabowski, op. cit., formula 10.6, pp 139-140.

[^15]:    ${ }^{29}$ Pratt and Grabowski, op. cit., Exhibit 10.6 summarizes guidance on when to use the various un-levering / re-levering formulas, p 128.
    ${ }^{30}$ Arthur G. Korteweg, "The Costs of Financial Distress across Industries," Working paper Stanford University (January 15, 2007): 65. Used with permission. All rights reserved. In today's market, debt betas have increased for even lower levels of leverage than displayed in this graph.

[^16]:    ${ }^{31}$ Ibid.
    ${ }^{32}$ Criteria for assignment to the high financial risk portfolio are: (1) companies in bankruptcy or liquidation;
    (2) companies with the 5 -year average net income or operating income in the prior 5 -years less than zero;
    (3) companies with negative book value of equity at any of the prior 5 fiscal year ends; or (4) companies with book value of debt to market value of equity greater than $80 \%$.

[^17]:    ${ }^{33}$ Pratt and Grabowski, op. cit., Appendix to Chapter 31 has a detailed example of these alternative analyses.

[^18]:    ${ }^{34}$ John R. Graham, "Debt and the Marginal Tax Rate," Journal of Financial Economics, (May 1996): 41-73.; Graham, "Proxies for the Corporate Marginal Tax Rate," Journal of Financial Economics, (October 1991): 187-221.; Graham and Michael Lemmon, "Measuring Corporate Tax Rate and Tax Incentives: A New Approach," Journal of Applied Corporate Finance (Spring 1998): 54-65.

[^19]:    ${ }^{35}$ John R. Graham and Lillian F. Mills, "Using tax return data to simulate corporate marginal tax rates," working paper (January 24, 2007).
    ${ }_{37}^{36}$ Pratt and Grabowski, op. cit., pp 297-308.
    ${ }^{37}$ Ignacio Velez-Pareja, "Return to Basics: Are you Properly Calculating Tax Shields?" November 30, 2008 available at http://ssrn.com/abstract=1306043 .

[^20]:    ${ }^{38}$ Benjamin Graham and David Dodd, Security Analysis (McGraw-Hill Professional) originally published in 1934; now in its $5^{\text {th }}$ ed. and authored by Sidney Cottle, Roger F. Murray, and Frank E. Block.

[^21]:    ${ }^{39}$ V.V. Chari, et al., op. cit., pp. 6-7.
    ${ }^{40}$ Duff \& Phelps' Risk Premium Report, op. cit., Exhibits D-1 and D-2 respectively can be used to estimate the COEC using the Build-up method.

[^22]:    ${ }^{41}$ Merrill Lynch Quantitative Profiles is published monthly.

[^23]:    ${ }^{42}$ In presenting one's cost of capital estimates, some analysts may want to separately identify the adjustments made due to temporary aberrations from the more long-term assumptions they typically use in their models; some analysts describe such adjustments as adding/subtracting an "alpha" from the estimate arrived at applying assumptions derived from data sources or in "standard" analyses. This article should help one quantify such adjustments.

